4th International Week at Óbuda University

Development and application of the hybrid multi-criteria decision making models in fuzzy environment

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Details about lecturer

- **Djordje Nikolić** holds Doctoral Degree from University of Belgrade in Engineering Management, and he received this scientific degree in the year of 2010. Since 2008, working at the University of Belgrade- Technical Faculty in Bor as an associate professor for subjects: Decision Theory, Management Information Systems, Management Systems and Quantitative methods.
- He is interested in applied management, especially in quantitative methods and multi-criteria decision theory. He is author or co-author of two books and several scientific papers: 22 papers have been published in SCI and SCIE journals, 11 papers have been published in national scientific journals, and over 30 papers are published on international and national symposiums.

Some important references

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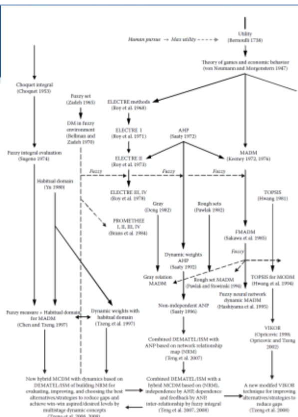
Learning objectives

- Understand the concept of multi-criteria decision making and how it differs from situations and procedures involving a single criterion
- Know how to apply the analytic hierarchy process (AHP) to solve a problem involving multiple criteria.
- Learn how to apply hybrid multi-criteria models to improve the analysis of the different management problems.
  - *An illustrative example: supplier prioritization in supply chain management*

Historical Development of Multi-Criteria (Multi-Objective) Decision Making

- Multiple criteria (objective) decision making is aimed at optimal design problems in which several (conflicting) criteria are to be achieved simultaneously.
- The characteristics of MCDM are a set of (conflicting) criteria and a set of well-defined constraints.



Decision Making Process

- "Decision making is the study of identifying and choosing alternatives based on the values and preferences of the decision maker. Making a decision implies that there are alternative choices to be considered, and in such a case we want not only to identify as many of these alternatives as possible but to choose the one that best fits with our goals, objectives, desires, values, and so on.." (Harris (1980))
- According to Baker et al. (2001), decision making should start with the identification of the decision maker(s) and stakeholder(s) in the decision, reducing the possible disagreement about problem definition, requirements, goals and criteria.

Harris, R. (1998) Introduction to Decision Making, VirtualSalt. http://www.virtualsalt.com/crebook5.htm
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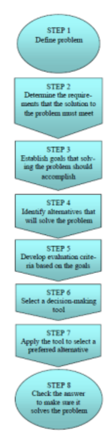


Analytic Hierarchy Process (AHP)

- AHP is one of the most popular multi-criteria methods developed by Thomas Saaty in 1980 (Saaty, 1980), as a method of solving socio-economic decision-making problems, and has been used to solve a wide range of decision-making problems.
- AHP is a multi- criteria decision making technique that can help express the general decision operation by decomposing a complicated problem into a multilevel hierarchical structure of objective, criteria and alternatives.
- AHP performs pairwise comparisons to derive relative importance of the variable in each level of the hierarchy and / or appraises the alternatives in the lowest level of the hierarchy in order to make the best decision among alternatives.
- What do we want to accomplish?
  - Learn how to conduct an AHP analysis
  - Understand the how it works
  - Deal with controversy
    - Rank reversal
    - Arbitrary ratings
  - Show what can be done to make it useable

Analytic Hierarchy Process (AHP)

- Many scientific papers have confirmed that the AHP method is very useful, reliable and systematic MCDM tool for solving complex decision problems (Kurttila et al., 2000; Kangas et al., 2001; Kajanusa et al., 2004; Lee et al., 2011).
- For example, the authors Vaidya and Kumar (2006) in their review work analyzed 27 papers, of about 150 papers cited in the references, pertaining to the application of the AHP method in various scientific fields.
- Furthermore, the AHP method allows pairwise comparisons between evaluation factors in order to determine the priorities among them, while using the approach of calculating eigenvalues (Gorener et al., 2012). Determination of the relative priority, when comparing pairs within the AHP methodology, is achieved by assigning an importance score according to the 1–9 scale of Saaty.

AHP methodology

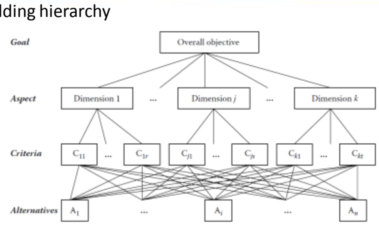
- Relationship between two elements that share a common parent in the hierarchy and numerical representation (Matrix)
- Comparisons ask 2 questions:
  - Which is more important with respect to the criterion?
  - How strongly?
- Matrix shows results of all such comparisons
- Typically uses a 1-9 scale
- Requires n(n-1)/2 judgments
- Inconsistency may arise

Saaty's 1-9 scale

Intensity of Importance	Definition
1	**Equal Importance**
3	**Moderate Importance**
5	**Strong Importance**
7	**Very Strong Importance**
9	**Extreme Importance**
2, 4, 6, 8	**For compromises between the above**
Reciprocals of above	**In comparing elements i and j** **- if i is 3 compared to j** **- then j is 1/3 compared to i**
Rationals	**Force consistency** **Measured values available** **CR≤0.1**

AHP steps

- Building hierarchy



- Collecting information i.e. performing pairwise comparison between elements
- Calculate eigenvector
- Results of synthesis

AHP steps

- To determine the importance of the criteria and sub-criteria, in this study, following steps of AHP method were conducted:
  - Defining pairwise comparison matrix A: after decomposition of the decision problem and forming of the hierarchical structure, the subsequent procedure for determining the relative importance of criteria pairs is based on the Saaty's scale 1 - 9.
  - For defined set of criteria within the appropriate level of the hierarchy C={Cj|j=1,2,..n}, results of a comparison of the elements at a given level of the hierarchy are placed in the appropriate pair-wise comparison matrix A (n x n). Each element a_{ij} (i,j=1,2,...n) of the matrix A can be defined as the quotient of the criteria weights:

$$A = (a_{ij})_{nxn} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 1/a_{12} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ 1/a_{1n} & 1/a_{2n} & \dots & a_{nn} \end{bmatrix}$$

  - The reciprocal value of the comparison results is placed in the position a_{ji}, where $a_{ji}=1/a_{ij}$ in order to maintain consistency. Thus, when i = j, then it follows that $a_{ij}=1$.

AHP steps

- Furthermore, if there is a perfectly consistent evaluation, matrix A could be shown in the following format:

$$A=(a_{ij})_{nxn}=\begin{bmatrix} w_1/w_1 & w_1/w_2 & \dots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \dots & w_2/w_n \\ \dots & \dots & \dots & \dots \\ w_n/w_1 & w_n/w_2 & \dots & w_n/w_n \end{bmatrix}$$

where w_i represents the relative weight coefficient of element i.

- Determination of weighting factors w_j:
  - Various methods have been proposed to extract values of vectors of the weight coefficients $w_j=\{w_1, \dots, w_n\}$ from the matrix A. Saaty (1980) suggested that for the matrix A its maximum eigenvalue λ_{max} should be determined first. The corresponding eigenvector of the matrix can then be taken as a vector of approximate values of the weight coefficients w_j, because the following applies:

$$A\cdot w=n\cdot w \quad \text{or} \quad \begin{bmatrix} w_1/w_1 & w_1/w_2 & \dots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \dots & w_2/w_n \\ \dots & \dots & \dots & \dots \\ w_n/w_1 & w_n/w_2 & \dots & w_n/w_n \end{bmatrix}\cdot\begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_n \end{bmatrix}=n\cdot\begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_n \end{bmatrix}$$

AHP steps

- If the matrix A is the completely consistent matrix, eigenvector w, which is a weight vector with $\sum_{j=1}^{n} w_j=1$, can be obtained by solving equation:

$$A\cdot w=n\cdot w \quad \text{or} \quad A\cdot w=\lambda_{max}\cdot w \quad \Rightarrow \quad (A-\lambda_{max}I)\cdot w=0$$

where λ_{max} is the maximum eigenvalue of the matrix A, while its rank is equal to 1, as well as λ_{max} = n, and I represents an identity matrix .

- In this case, the values of the vectors of the weight coefficients $w_j=\{w_1, \dots, w_n\}$ can be obtained by normalizing either rows or columns of the matrix A (Gorener et al., 2012).

AHP test of consistency

- AHP is a popular method because it has the ability to identify and analyze the inconsistency of decision-makers' judgments in the process of discernment and valuation of the elements of the hierarchy (Chang and Huang, 2006). If the values of the weight coefficients of all the elements that are mutually compared at a given level of the hierarchy could be precisely determined, the eigenvalues of the matrix A would be entirely consistent. However, that is relatively difficult to achieve in practice. AHP method provides the ability to measure errors of judgment by computing consistency index (CI) for the obtained comparison matrix A, and then calculating the consistency ratio (CR).
- In order to calculate the consistency ratio (CR), we first need to calculate the consistency index (CI) according to the following formula:

$$CI=\frac{\lambda_{max}-n}{n-1}$$

- Next, the consistency ratio is determined by equation:

$$CR=\frac{CI}{RI}$$

- where RI is the random index which depends on the order n of the matrix A

AHP test of consistency

- also, λ_{max} is maximal eigenvalue of the matrix A:

$$\begin{bmatrix} a_{11} & a_{12} & .. & a_{1n} \\ a_{21} & a_{22} & .. & a_{2n} \\ . & . & . & . \\ . & . & . & . \\ a_{n1} & a_{n2} & .. & a_{nn} \end{bmatrix} * \begin{bmatrix} w_1 \\ w_2 \\ . \\ . \\ w_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ b_n \end{bmatrix} \rightarrow \begin{bmatrix} \frac{b_1}{w_1} \\ \frac{b_2}{w_2} \\ . \\ . \\ \frac{b_n}{w_n} \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ . \\ . \\ \lambda_n \end{bmatrix} \rightarrow \lambda_{max}=\frac{1}{n}\sum_{i=1}^{n}\lambda_i$$

- Random indices (RI)

n-order of the matrix A	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

Example: Gill Glass

- Designer Gill Glass must decide which of three manufacturers will develop his "signature" toothbrushes. Three factors are important to Gill:

(1) his costs;

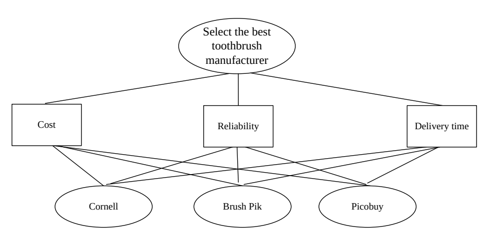
(2) reliability of the product; and,

(3) delivery time of the orders.

- The three manufacturers are Cornell Industries, Brush Pik, and Picobuy. Cornell Industries will sell toothbrushes to Gill Glass for $100 per gross, Brush Pik for $80 per gross, and Picobuy for $144 per gross.

Example: Gill Glass

- Hierarchical structure of the selection problem



Example: Gill Glass

- Pairwise comparison matrix: Criteria

	Cost	Reliability	Delivery
Cost	1	7	9
Reliability	1/7	1	7
Delivery	1/9	1/7	1

- Divide each entry in the pair-wise comparison matrix by its corresponding column sum.

	Cost	Reliability	Delivery
Cost	63/79	49/57	9/17
Reliability	9/79	7/57	7/17
Delivery	7/79	1/57	1/17

Example: Gill Glass

- The priority vector for Criteria relative to the primary goal is determined by averaging the row entries in the normalized matrix. Converting to decimals we get:

Cost: (63/79 + 49/57 + 9/17)/3 = .729
Reliability: (9/79 + 7/57 + 7/17)/3 = .216
Delivery: (7/79 + 1/57 + 1/17)/3 = .055

Example: Gill Glass

- Pairwise comparison matrix: Cost

	Cornell	Brush Pik	Picobuy
Cornell	1	1/3	6
Brush Pik	3	1	7
Picobuy	1/6	1/7	1

- Divide each entry in the pair-wise comparison matrix by its corresponding column sum.

	Cornell	Brush Pik	Picobuy
Cornell	6/25	7/31	6/14
Brush Pik	18/25	21/31	7/14
Picobuy	1/25	3/31	1/14

Example: Gill Glass

- The priority vector for the criterion Cost is determined by averaging the row entries in the normalized matrix. Converting to decimals we get:

Cornell: (6/25 + 7/31 + 6/14)/3 = .298
Brush Pik: (18/25 + 21/31 + 7/14)/3 = .632
Picobuy: (1/25 + 3/31 + 1/14)/3 = .069

Example: Gill Glass

- Pairwise comparison matrix: Reliability

	Cornell	Brush Pik	Picobuy
Cornell	1	7	2
Brush Pik	1/7	1	5
Picobuy	1/2	1/5	1

- Divide each entry in the pair-wise comparison matrix by its corresponding column sum.

	Cornell	Brush Pik	Picobuy
Cornell	14/23	35/41	2/8
Brush Pik	2/23	5/41	5/8
Picobuy	7/23	1/41	1/8

Example: Gill Glass

- The priority vector for the criterion Reliability is determined by averaging the row entries in the normalized matrix. Converting to decimals we get:

Cornell: (14/23 + 35/41 + 2/8)/3 = .571
Brush Pik: (2/23 + 5/41 + 5/8)/3 = .278
Picobuy: (7/23 + 1/41 + 1/8)/3 = .151

Example: Gill Glass

- Pairwise comparison matrix: Delivery Time

	Cornell	Brush Pik	Picobuy
Cornell	1	8	1
Brush Pik	1/8	1	1/8
Picobuy	1	8	1

- Divide each entry in the pair-wise comparison matrix by its corresponding column sum.

	Cornell	Brush Pik	Picobuy
Cornell	8/17	8/17	8/17
Brush Pik	1/17	1/17	1/17
Picobuy	8/17	8/17	8/17

Example: Gill Glass

- The priority vector for the criterion Delivery Time is determined by averaging the row entries in the normalized matrix. Converting to decimals we get:

Cornell: (8/17 + 8/17 + 8/17)/3 = .471
Brush Pik: (1/17 + 1/17 + 1/17)/3 = .059
Picobuy: (8/17 + 8/17 + 8/17)/3 = .471

Example: Gill Glass

The overall priorities are determined by multiplying the priority vector of the criteria by the priorities for each decision alternative for each objective.

Priority Vector
for Criteria [.729 .216 .055]

	Cost	Reliability	Delivery
Cornell	.298	.571	.471
Brush Pik	.632	.278	.059
Picobuy	.069	.151	.471

Example: Gill Glass

Thus, the overall priority vector is:

Cornell: (.729)(.298) + (.216)(.571) + (.055)(.471) = .366
Brush Pik: (.729)(.632) + (.216)(.278) + (.055)(.059) = **.524**
Picobuy: (.729)(.069) + (.216)(.151) + (.055)(.471) = .109

Brush Pik appears to be the overall recommendation.