



Application of statistical tools and techniques in Quality Management

Asst. Professor Dr Predrag Djordjevic

**University of Belgrade, Technical Faculty in Bor,
Serbia**

QUALITY IN SOCIETY

- The concept was known back in the primordial human communities
- It's original role was to ensure the survival, later it became a matter of social status
- In early communities, a buyer was usually responsible for "quality assurance"
- Industrial Revolution: rise of inspection and separate quality departments



DEVELOPMENT OF QUALITY IN USA

- Industry was under the influence of the Taylor's theory – quality was independent from design and production
- 1924 - Bell Telephone Laboratories
- 1931 - Shewhart published the first book in the field of quality control (500 pages)
- World War II



DEVELOPMENT OF QUALITY IN USA



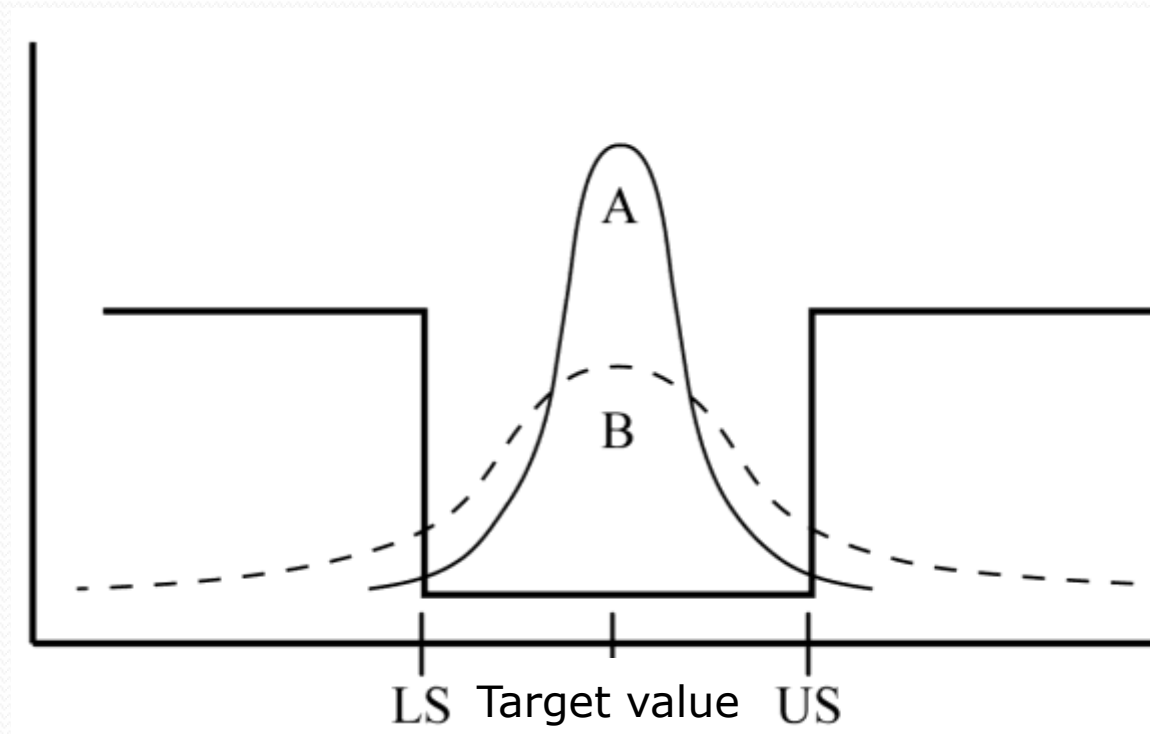
- Deming is training workers and raising the level of industrial production up to the level of hostile countries competing in the World War II

QUALITY IN JAPAN

- Japan was on the losing side at the end of World War II
- Traditionally agricultural economy in addition to the strong military industry
- Gen. MacArthur invites US experts for the theory of quality to educate the Japanese leaders as a form of post-war help
- Deming introduces the concept of SPC to Japanese managers
- Japanese economic miracle



New approach to Quality using SPC

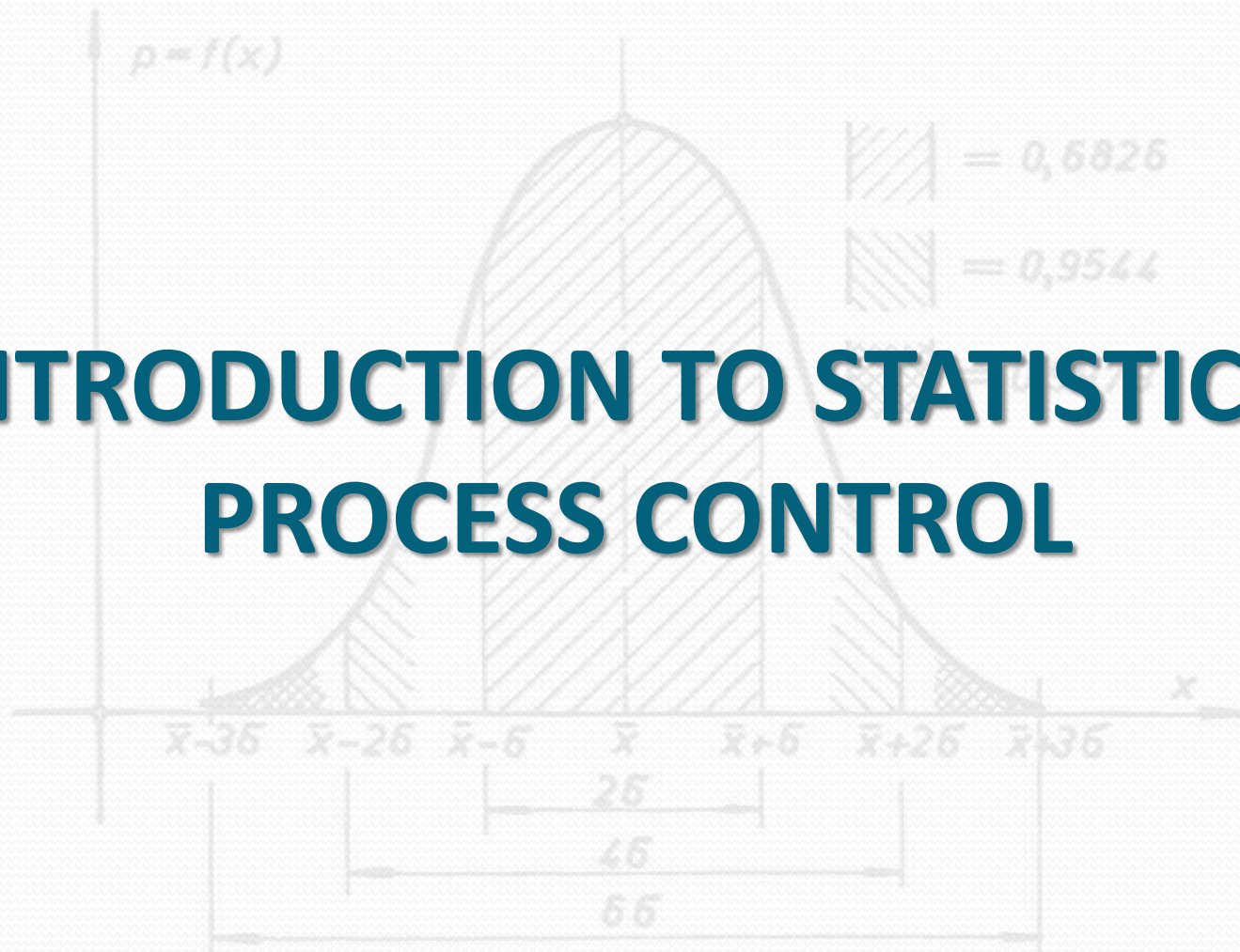


Traditional vs. new approach to Quality Management using SPC

Formal Definitions of Quality

- The totality of features and characteristics of a product or service that bears on its ability to satisfy given needs – American Society for Quality
- Other definitions:
 - ✓ Fitness for use
 - ✓ Conformance to specifications
 - ✓ Meeting or exceeding customer expectations

INTRODUCTION TO STATISTICAL PROCESS CONTROL



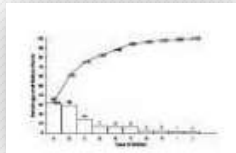
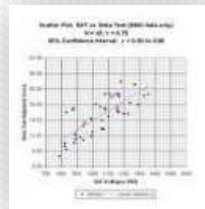
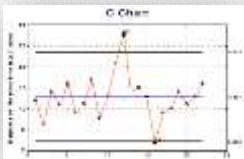
STATISTICAL PROCESS CONTROL (SPC)

- To meet or exceed customer expectations product should be produced by a process that is **stable or repeatable**
- The process must be capable of operating with little **variability** around the target **dimensions** of the product's quality characteristics
- SPC is a powerful collection of problem-solving tools useful in achieving process stability and improving capability through the reduction of variability
- Elimination of waste is another key element of SPC

7 FUNDAMENTAL QUALITY TOOLS

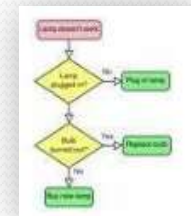
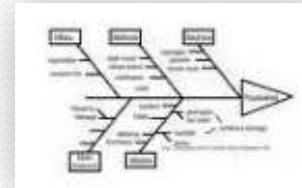
Tools for numerical data

- Control chart
- Histogram (stem-and-leaf plot)
- Pareto chart
- Scatter diagram



Tools for non-numerical data

- Check sheet
- Cause-and-effect diagram
- Flowchart



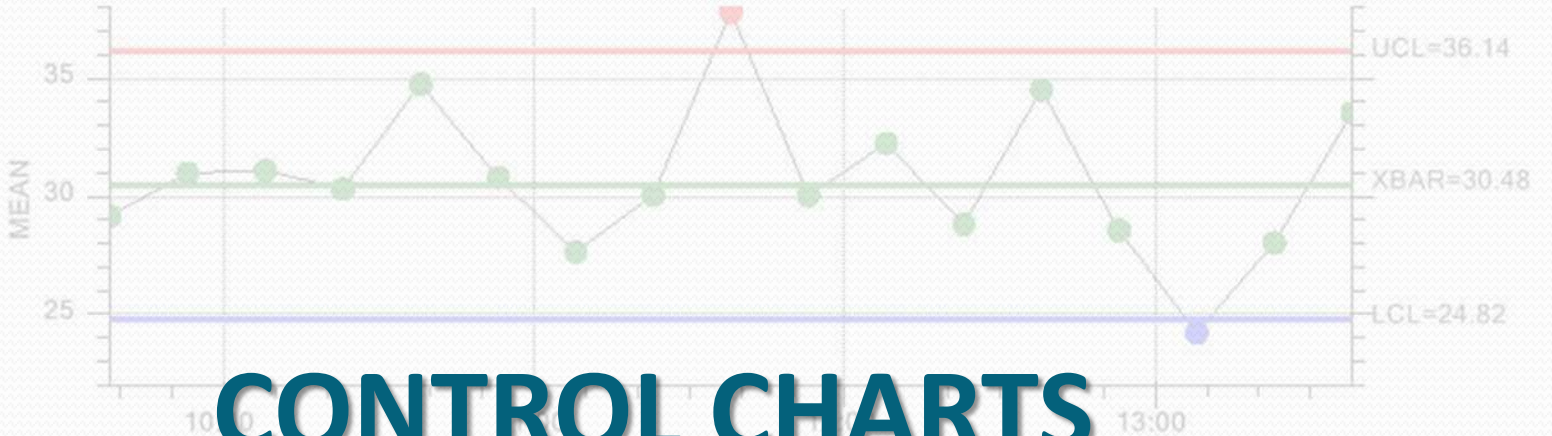
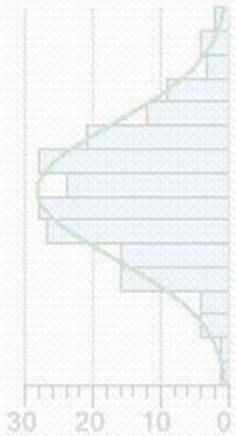
Title: Variable Control Chart (X-Bar & R)

Part No.: 283501

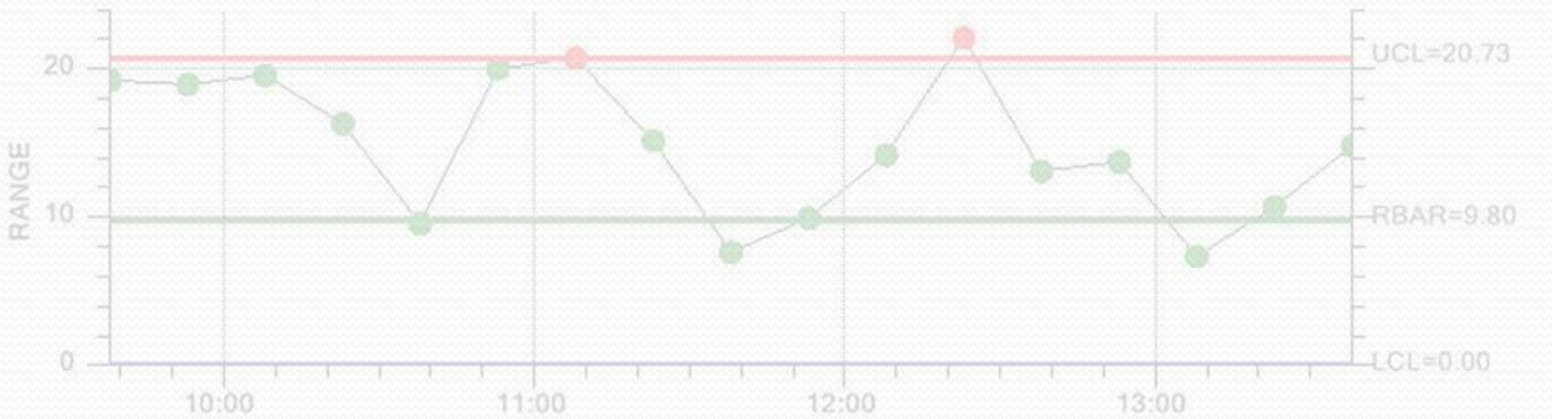
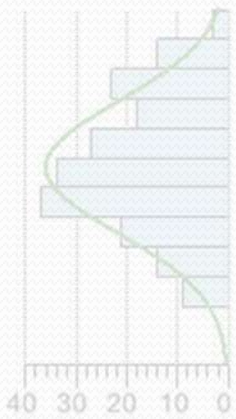
Chart No.: 17

Date: 5/2/2008 11:53:01 AM

ALARM



CONTROL CHARTS



Control charts

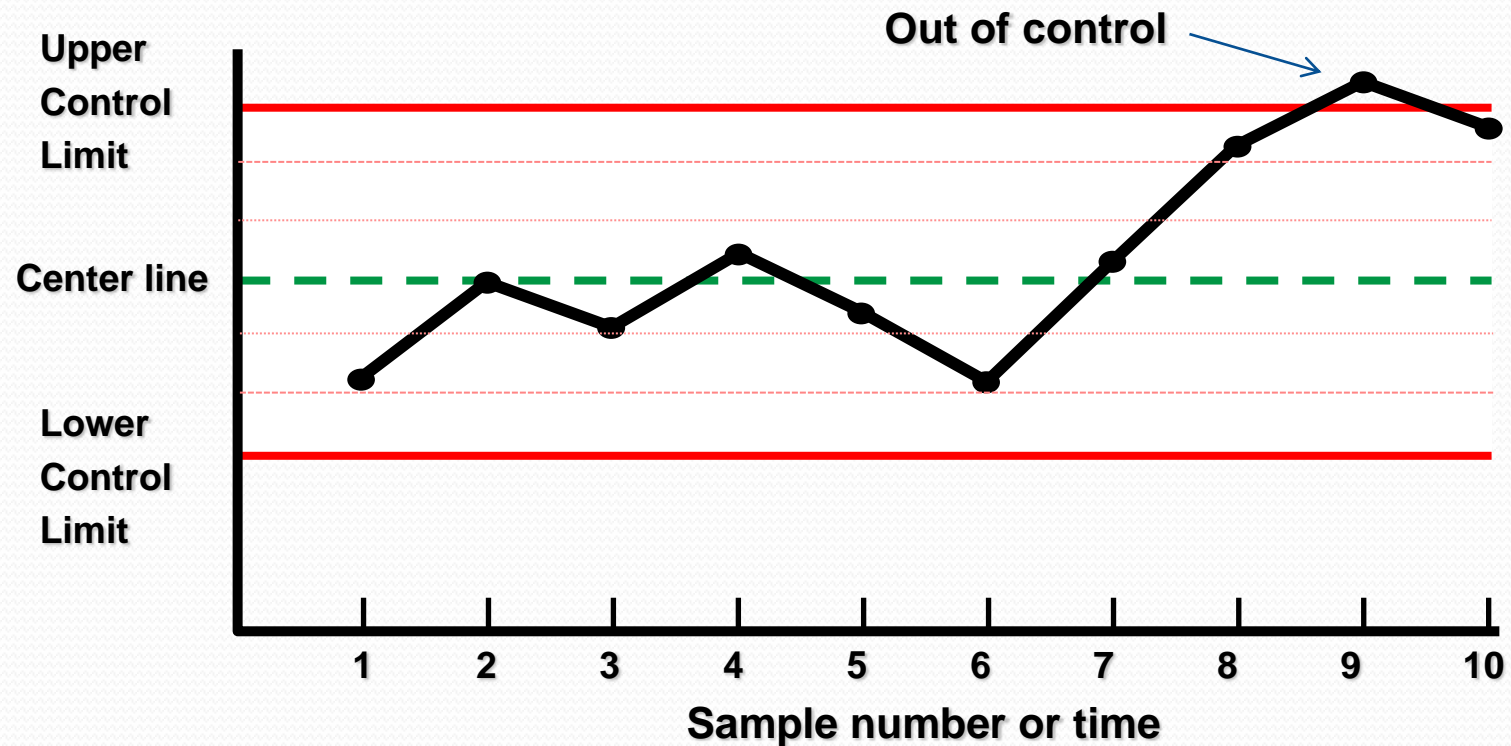


- Graphical representation of a quality characteristic that has been measured or computed from a sample vs. the sample number or time
- Control charts can be used to determine if a process has been in a state of statistical control by examining past data (Stage 1)
- They can be used to determine control limits (CL) that would apply to future data obtained from a process, in order to determine if the process is being maintained in a state of statistical control (Stage 2)
- They are used to track variability in key quality characteristics

Control charts



- Graphical representation of a quality characteristic that has been measured or computed from a sample vs. the sample number or time



Variability in the process

• Natural

- random, common causes
- inherent in every process
- can be eliminated only through systematic improvements

• Assignable

- specific identifiable causes:
 - machines
 - operator errors
 - defective raw material
- they can be managed

Types of Control Charts

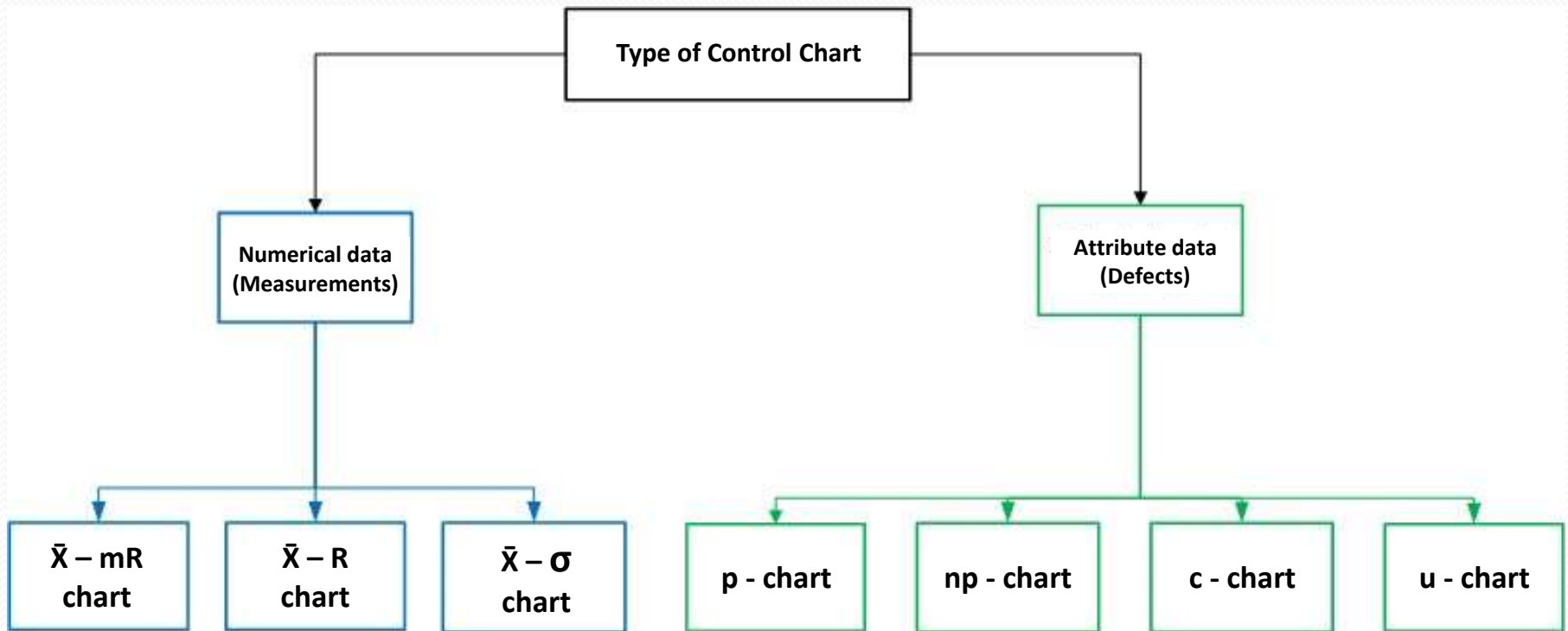
- **Variables**

- Quality characteristics of the products which can be measured on a numerical scale
 - length or width, diameter, temperature, volume etc.

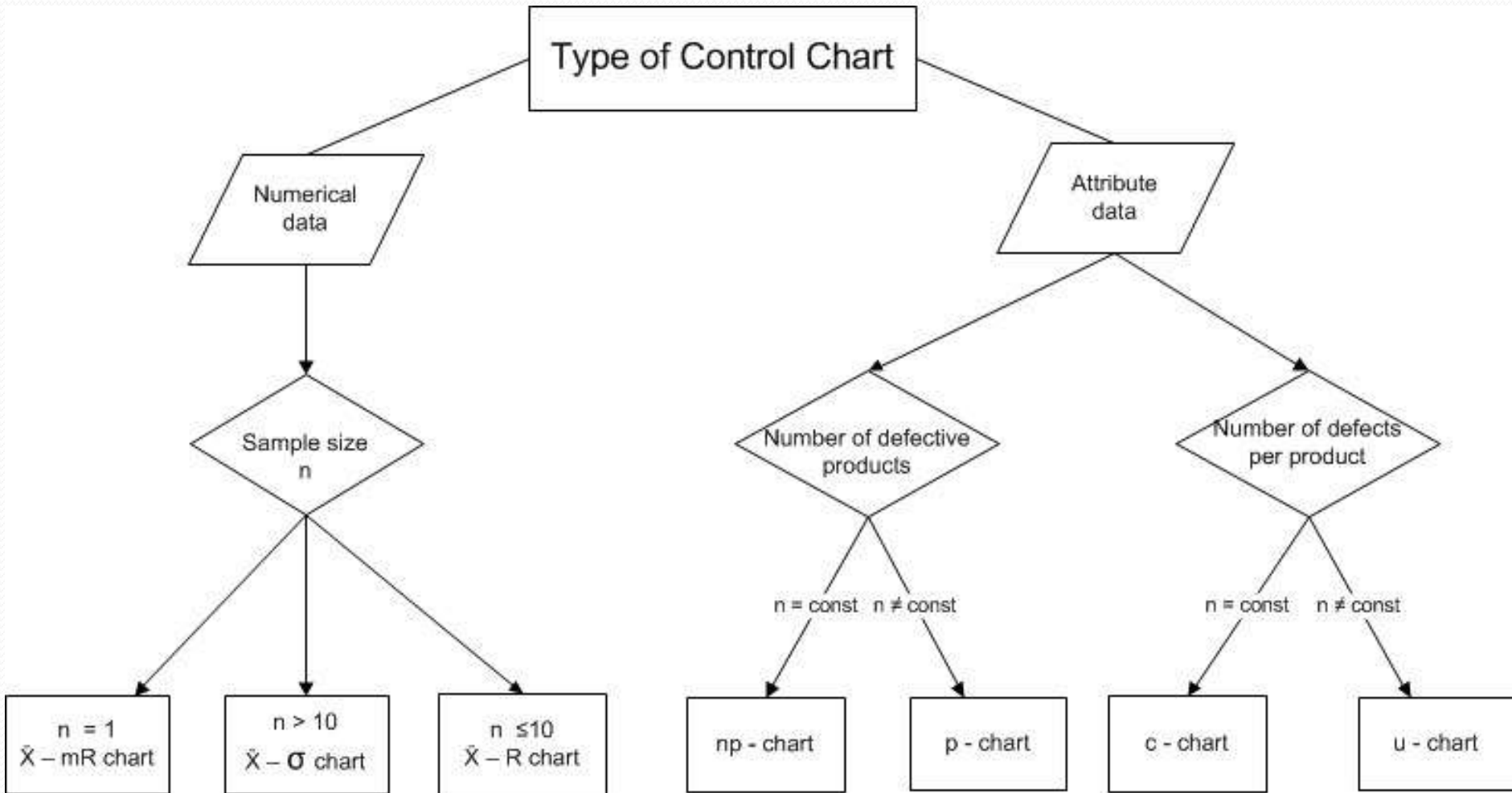
- **Attributes**

- Quality characteristics which can be described with discrete data
 - conforming or nonconforming, defective or nondefective

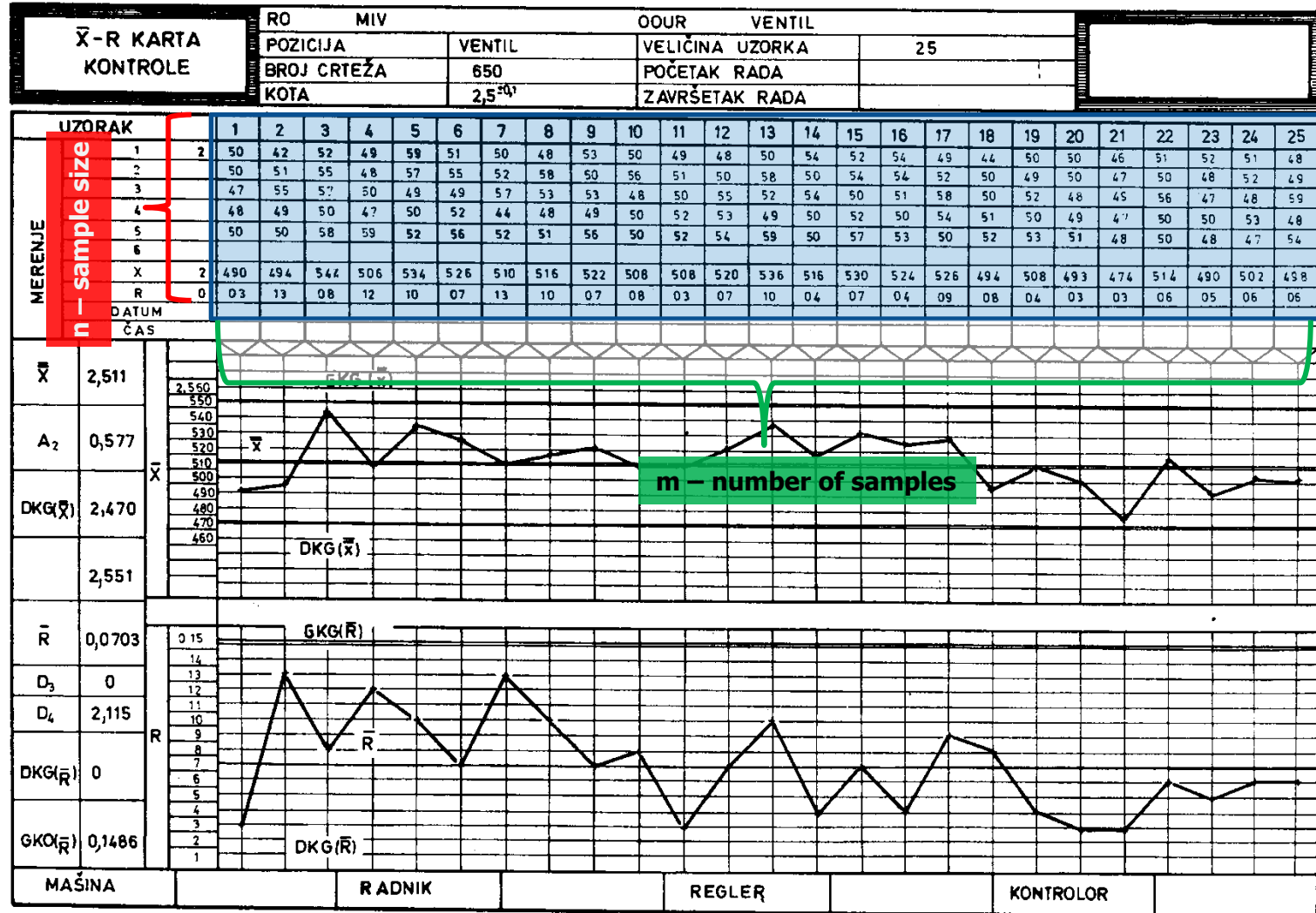
Types of Control Charts



Choosing the right type of Control Chart



Elements of the Control Chart



Elements of the Control Chart

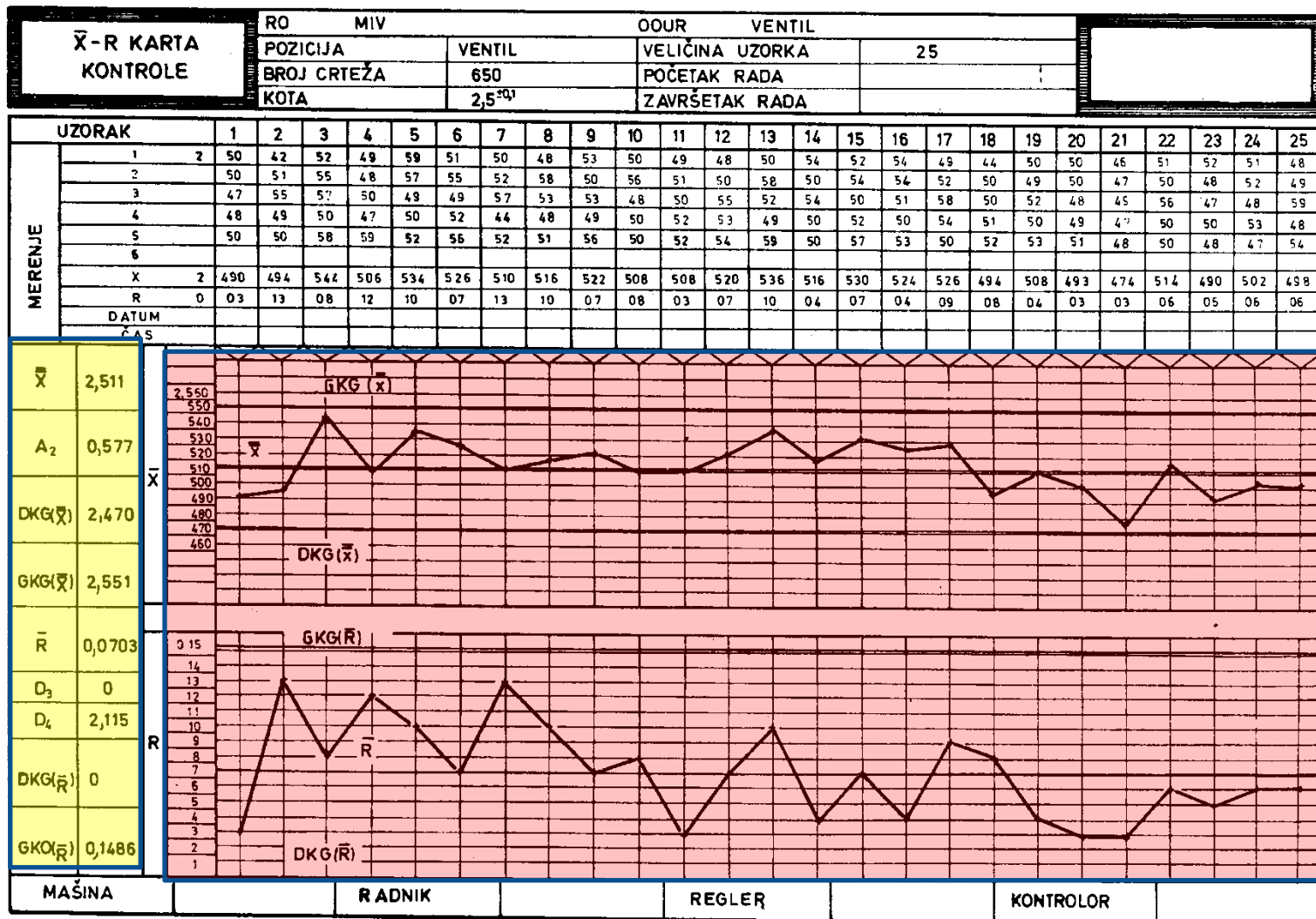


Tabela 3.

Faktori za centralne linije i kontrolne granice za proteklu proizvodnju								
Veličina uzorka n	A ₁	A ₂	B ₃	B ₄	D ₃	D ₄	d ₂	c ₂
2	3,760	1,880	0	3,267	0	3,267	1,128	0,5642
3	2,394	1,023	0	2,568	0	2,575	1,693	0,7236
4	1,880	0,729	0	2,266	0	2,282	2,059	0,7979
5	1,596	0,577	0	2,089	0	2,115	2,326	0,8407
6	1,410	0,483	0,030	1,970	0	2,004	2,534	0,8686
7	1,277	0,419	0,118	1,882	0,076	1,924	2,704	0,8882
8	1,175	0,373	0,185	1,815	0,136	1,864	2,847	0,9027
9	1,094	0,337	0,239	1,761	0,184	1,816	2,970	0,9139
10	1,028	0,308	0,284	1,716	0,223	1,777	3,078	0,9227
11	0,973	0,285	0,321	1,679	0,256	1,744	3,173	0,9300
12	0,925	0,266	0,354	1,646	0,284	1,716	3,258	0,9358
13	0,884	0,249	0,382	1,618	0,308	1,692	3,336	0,9410
14	0,848	0,235	0,405	1,594	0,329	1,671	3,407	0,9453
15	0,816	0,223	0,428	1,572	0,348	1,652	3,472	0,9490
16	0,788	0,212	0,448	1,552	0,364	1,636	3,532	0,9523
17	0,762	0,203	0,466	1,534	0,379	1,621	3,588	0,9551
18	0,738	0,194	0,482	1,518	0,392	1,608	3,640	0,9576
19	0,717	0,187	0,497	1,503	0,404	1,596	3,689	0,9599
20	0,697	0,180	0,510	1,590	0,414	1,586	3,735	0,9619
21	0,679	0,173	0,523	1,477	0,425	1,575	3,778	0,9638
22	0,662	0,167	0,534	1,466	0,434	1,566	3,818	0,9655
23	0,647	0,162	0,545	1,455	0,443	1,557	3,858	0,9670
24	0,632	0,157	0,555	1,445	0,452	1,548	3,895	0,9684
25	0,619	0,153	0,565	1,435	0,459	1,541	3,931	0,9696
Preko 25	$\frac{3}{\sqrt{n}}$		$1 - \frac{3}{\sqrt{2n}}$	$1 + \frac{3}{\sqrt{2n}}$				1

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

$$R = X_{\max} - X_{\min}$$

$$UCL_{(\bar{X})} = \bar{\bar{X}} + A_2(n) \cdot \bar{R}$$

$$LCL_{(\bar{X})} = \bar{\bar{X}} - A_2(n) \cdot \bar{R}$$

$$UCL_{(\bar{R})} = D_3 \bar{R}$$

$$LCL_{(\bar{R})} = D_4 \bar{R}$$

$$LCL(\bar{x}) = 2,511 - 0,577 \cdot 0,070 = 2,470 \text{ mm}$$

$$UCL(\bar{x}) = 2,511 + 0,577 \cdot 0,070 = 2,511 \text{ mm}$$

$$LCL(\bar{R}) = 0 \cdot 0,070 = 0 \text{ mm}$$

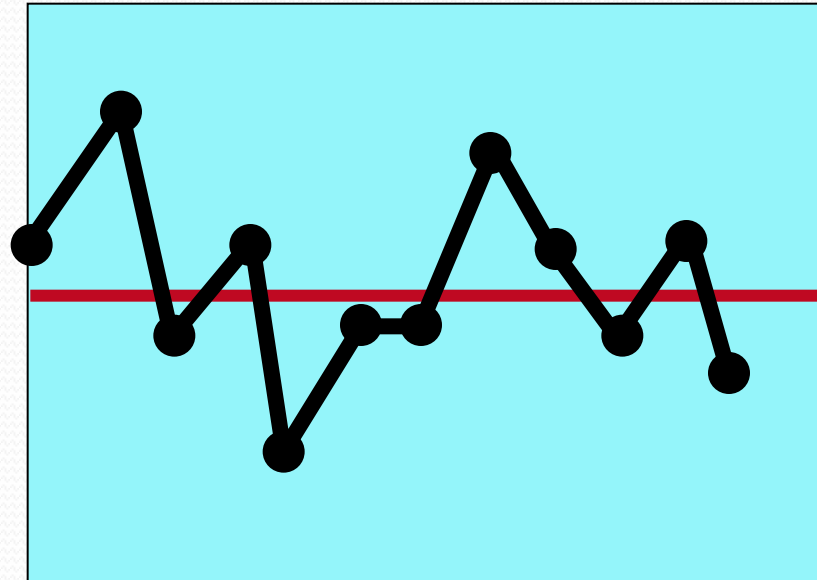
$$UCL(\bar{R}) = 2,115 \cdot 0,070 = 0,148 \text{ mm}$$

Patterns in Control Charts

Upper control limit

Target

Lower control limit



Normal behavior

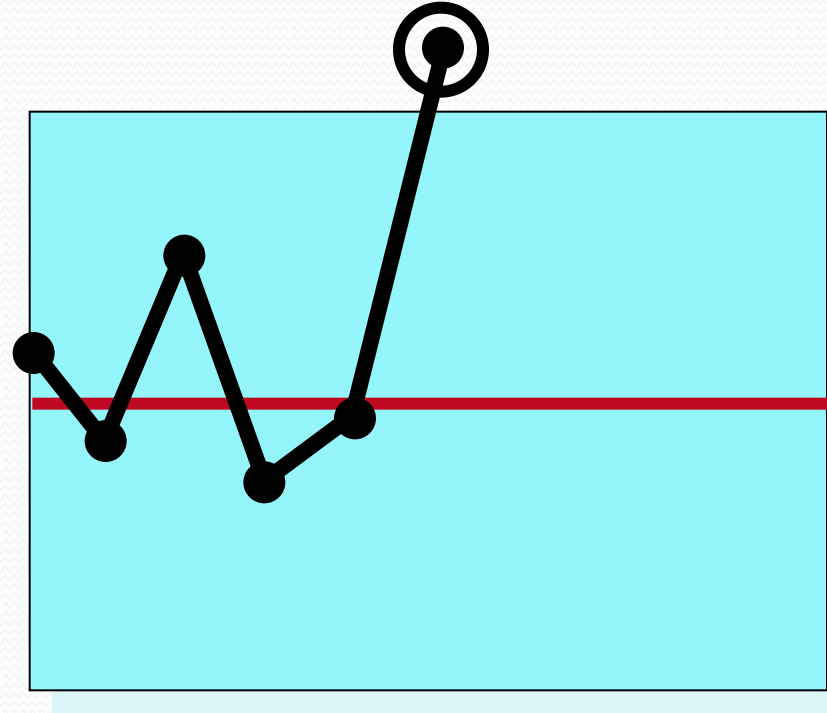
Process is "in control"

Patterns in Control Charts

Upper control limit

Target

Lower control limit



One plot out above (or below)

Investigate for cause

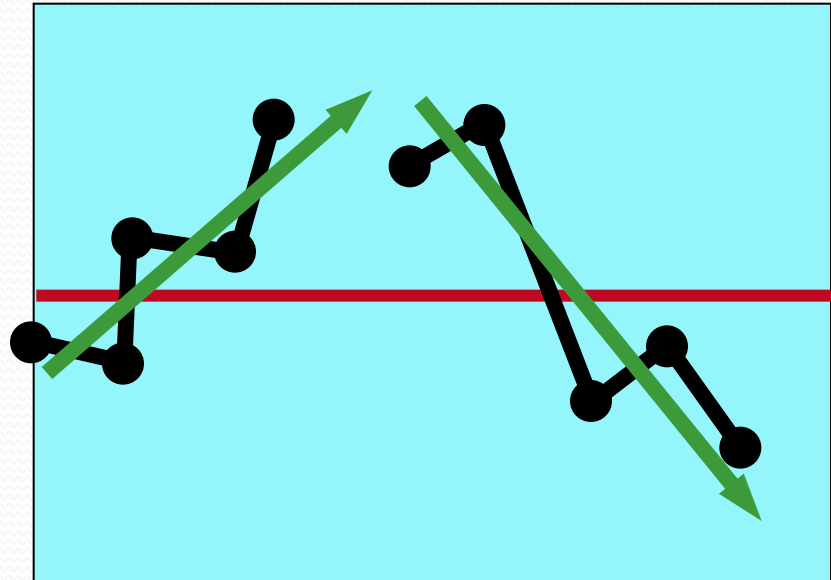
Process is "out of control."

Patterns in Control Charts

Upper control limit

Target

Lower control limit



Trends in either direction, 5 plots

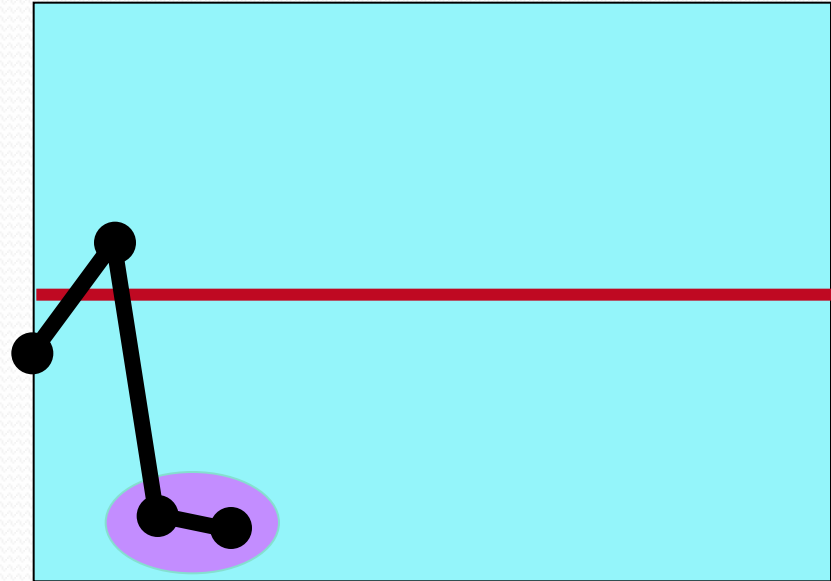
Investigate for cause of progressive change

Patterns in Control Charts

Upper control limit

Target

Lower control limit



Two plots very near lower (or upper) control

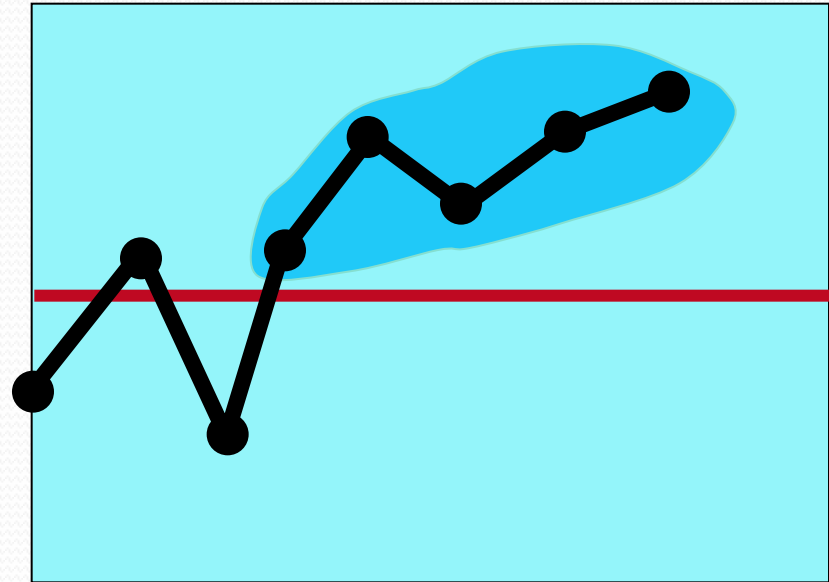
Investigate for cause

Patterns in Control Charts

Upper control limit

Target

Lower control limit



**Run of 5 above (or below)
central line**

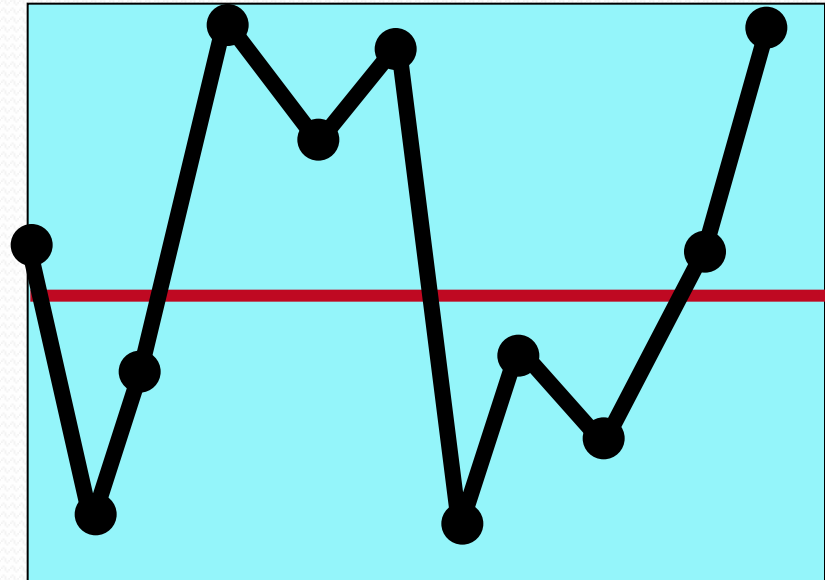
Investigate for cause

Patterns in Control Charts

Upper control limit

Target

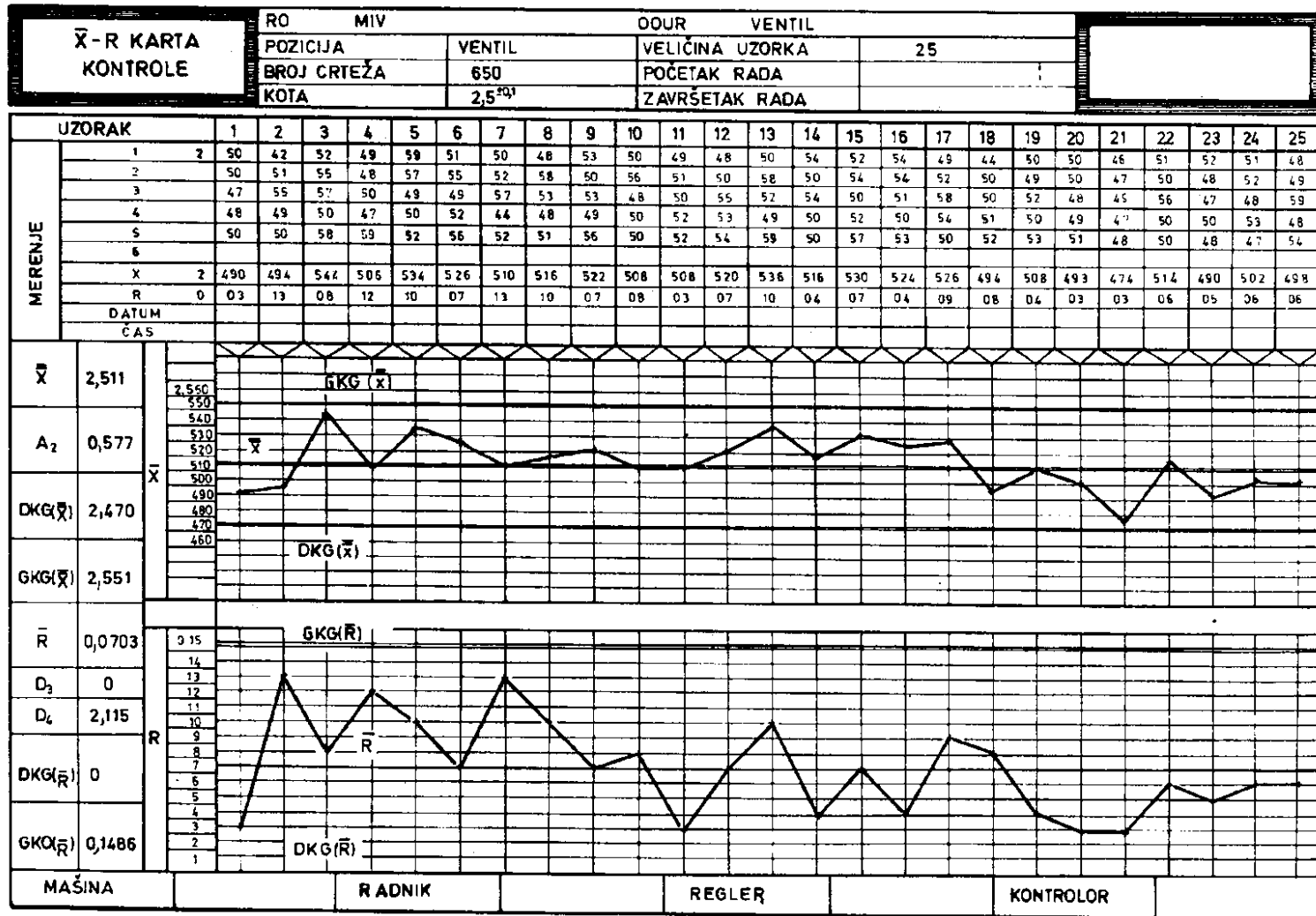
Lower control limit



Erratic behavior

Investigate

Interpretation of Control Charts



The process is under statistical control because all points fall within the UCL and LCL

X - σ Control Chart

- When the sample size in the series is $n > 10$
- Determination of control limits:

$$\bar{\bar{X}} = \frac{1}{n} \sum_{i=1}^n X_i$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

$$UCL(\bar{\bar{X}}) = \bar{\bar{X}} + A_1(n) \cdot \bar{\sigma}$$

$$UCL(\bar{\sigma}) = B_4(n) \cdot \bar{\sigma}$$

$$LCL(\bar{\bar{X}}) = \bar{\bar{X}} - A_1(n) \cdot \bar{\sigma}$$

$$LCL(\bar{\sigma}) = B_3(n) \cdot \bar{\sigma}$$

$(\bar{X} - \sigma)$ KARTA
KONTROLE

PREDUZEĆE

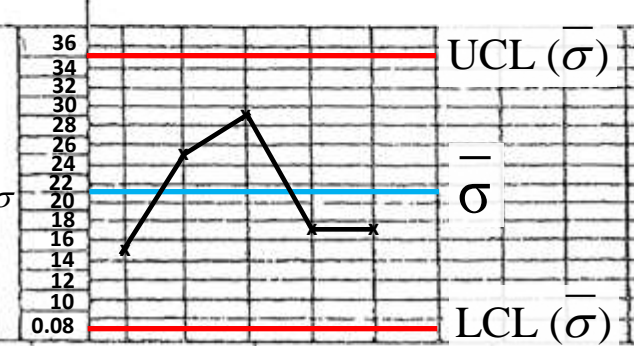
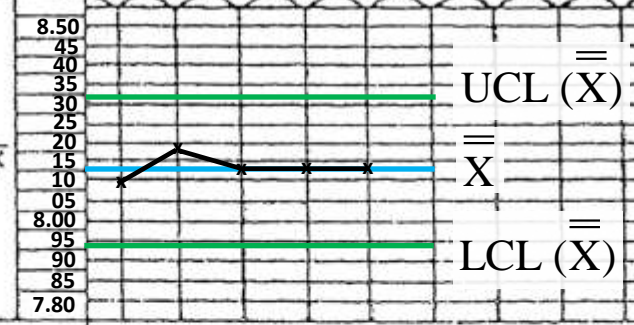
POZICIJA
BROJ CRTEŽA
KOTA

VELIČINA UZORKA
POČETAK RADA
ZAVRŠETAK RADA

M
E
R
E
N
J
E

UZORAK	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	7.90	8.10	8.50	8.30	8.25																				
2	8.50	8.50	8.10	8.50	8.50																				
3	8.20	8.20	8.20	8.20	8.20																				
4	8.10	8.90	8.00	8.00	8.00																				
5	8.00	8.00	7.90	7.90	7.90																				
6	8.10	8.30	8.30	8.00	8.20																				
7	8.00	8.00	8.50	8.25	8.25																				
8	8.00	8.10	8.00	8.00	8.00																				
9	8.20	8.10	8.00	8.00	8.10																				
10	8.20	8.00	8.00	8.10	7.90																				
11	7.95	8.00	8.10	8.00	8.00																				
12	8.00	8.10	8.00	8.30	8.25																				
13																									
14																									
15																									
\bar{X}	8.09	8.19	8.13	8.13	8.13																				
σ	0.15	0.25	0.29	0.17	0.17																				
DAJUM ČAS																									

\bar{X}	8.13
A_1	0.925
$LCL(\bar{X})$	7.94
$UCL(\bar{X})$	8.32
σ	0.21
B_3	0.354
B_4	1.646
$LCL(\sigma)$	0.07
$UCL(\sigma)$	0.35



The process is under statistical control because all points fall within the UCL and LCL

Problem

In the factory for the manufacturing of plastic packaging Uniplast a sample is taken hourly, consisting of a number of plastic bottles which are tested for defects. Each bottle on which a defect would be found is being marked as nonconforming and sent for recycling. The table lists the number of bottles inspected during one shift and the number of defective bottles. Construct an appropriate control chart and assess the stability of the process.

No. of sample series (m)	No. of tested bottles (n_i)	No. of defective bottles (d_{ni})
1	24	1
2	28	2
3	36	4
4	57	4
5	52	8
6	40	12
7	44	7
8	46	5

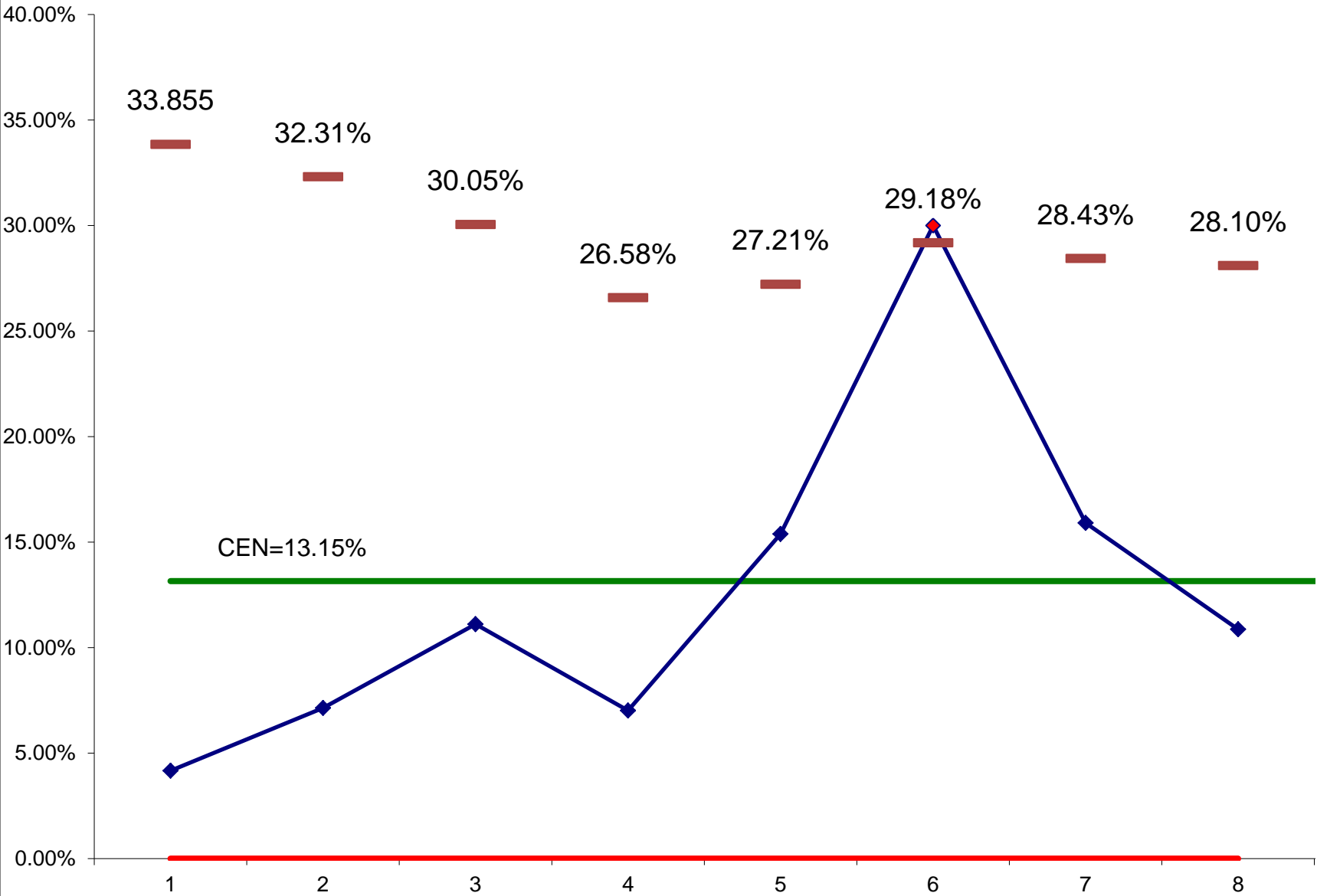
p – Chart

- Control chart for monitoring fraction of nonconforming units
- The percentage of defective products is calculated as $p(\%) = \frac{d}{n} \cdot 100$
- The mean value of the percentage of defective units - the central line

$$\bar{p} = 100 \cdot \frac{\text{total number of defective products in all samples}}{\text{total number of sampled products}} = 100 \cdot \frac{D}{N} = 100 \cdot \frac{\sum_{i=1}^m d_i}{n \cdot m}$$

$${}^U_L CL(\bar{p}) = \bar{p} \pm 3 \cdot \sqrt{\frac{\bar{p}(100 - \bar{p})}{n}}$$

p Chart



Problem

In a facility belonging to the Canon company, in the production line for image sensors, a strict testing procedure requires sampling of 25 chips each hour and running a number of tests in order to determine the output quality of the chips. If any nonconformity is detected on the sensor, it is considered as defective. The table presents testing results. Construct an appropriate control chart and assess the stability of the process.

No. of samples (k)	No. of tested sensors (n)	No. of defective sensors in the sample (m)
1	25	3
2	25	7
3	25	4
4	25	7
5	25	11
6	25	5
7	25	3
8	25	4
9	25	2
10	25	1

np – Chart

- When dealing with number of defective products in the sample
- When testing samples of the same size

$$\overline{np} = \bar{m} = \frac{\sum_{i=1}^m m_i}{k}$$

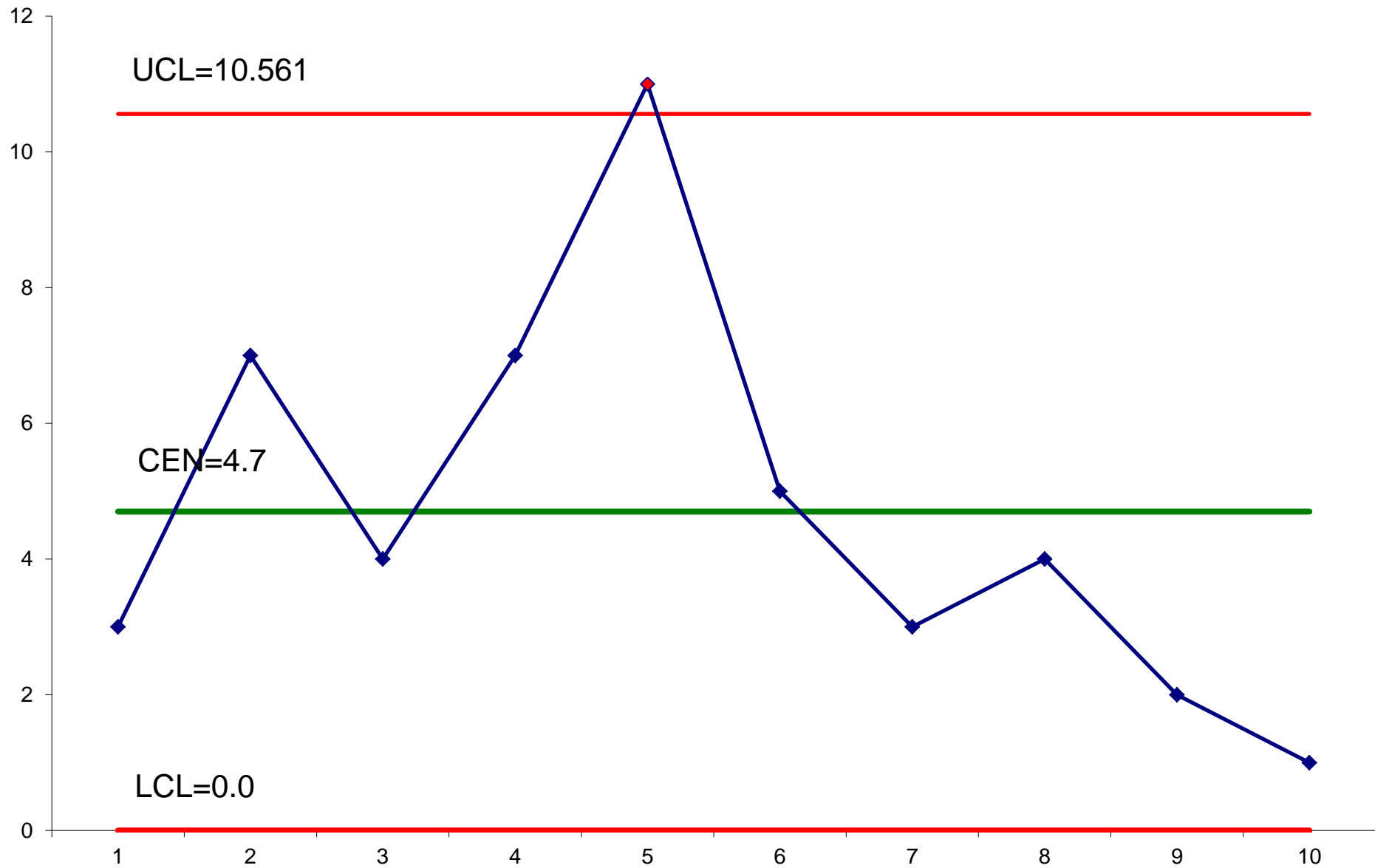
m_i – number of defective products within a single sample

k – total number of series in the sample

$${}^U_L CL(\bar{m}) = \bar{m} \pm 3 \cdot \sqrt{\bar{m} \cdot \left(1 - \frac{M}{N}\right)}$$

N - total number of sampled products ($k \cdot n$)

np Chart



Problem

In the branch facility of the firm U. S. Steel in Serbia, in a plant's tinplate production line, the quality of produced tinplate is controlled by collecting the 30x30 cm of sheet samples. These samples are then tested for defects. The number of defects found in each of the samples are shown in the table. Construct an appropriate control chart and assess the stability of this manufacturing process.

Sample No. (k)	No. of defects in the sample (d)
1	3
2	8
3	4
4	2
5	1
6	2
7	4
8	7
9	3
10	3

c – Chart

- Control of the number of defects per sample, when there is no limit to the number of defects that can be found in one product
- When testing samples of the same size

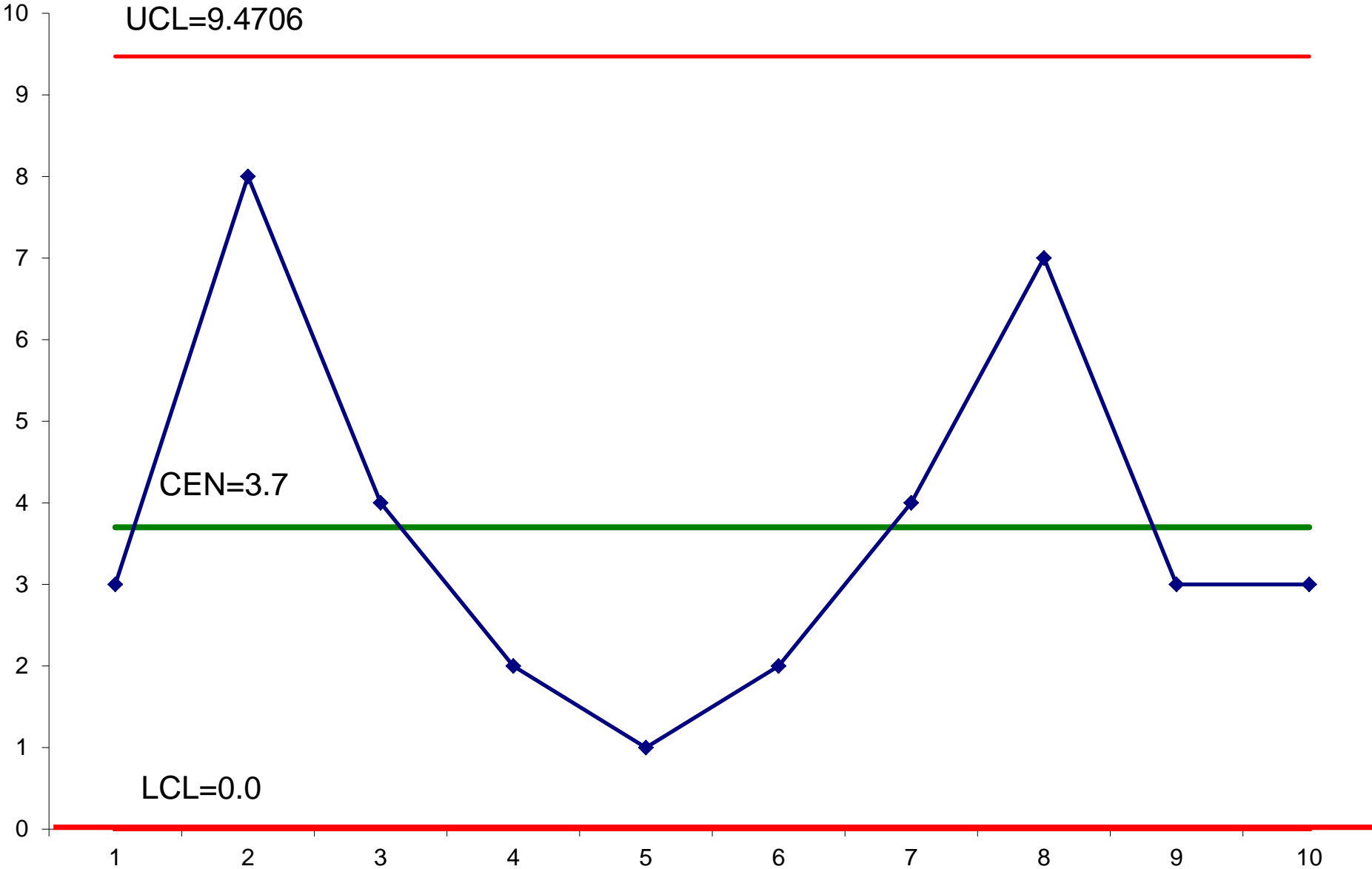
$$\bar{c} = \frac{\sum_{i=1}^k d_i}{k}$$

d_i - number of defects within a single sample

k – total number of samples

$${}^U_L CL_{(\bar{c})} = \bar{c} \pm 3 \cdot \sqrt{\bar{c}}$$

c - Chart



Problem

In order to examine the quality of the manufactured chips in one of the subcontracting manufacturing facilities of the company AMD, which produces chips for computer graphics cards, a number of manufactured chips are randomly sampled from each batch and thoroughly examined. Any irregularity regarding the number of defective Stream processors is registered and their number, along with a corresponding number of sampled units is presented in table. Construct an appropriate control chart and assess the stability of the process.

Series No. (m)	No. of tested chips in the series (n)	No. of defects in the sample (d)
1	5	22
2	7	19
3	4	23
4	6	27
5	8	31
6	10	28

u – Charts

- Control of the number of defects per sample, when there is no limit to the number of defects that can be found in one product
- When testing samples of different size

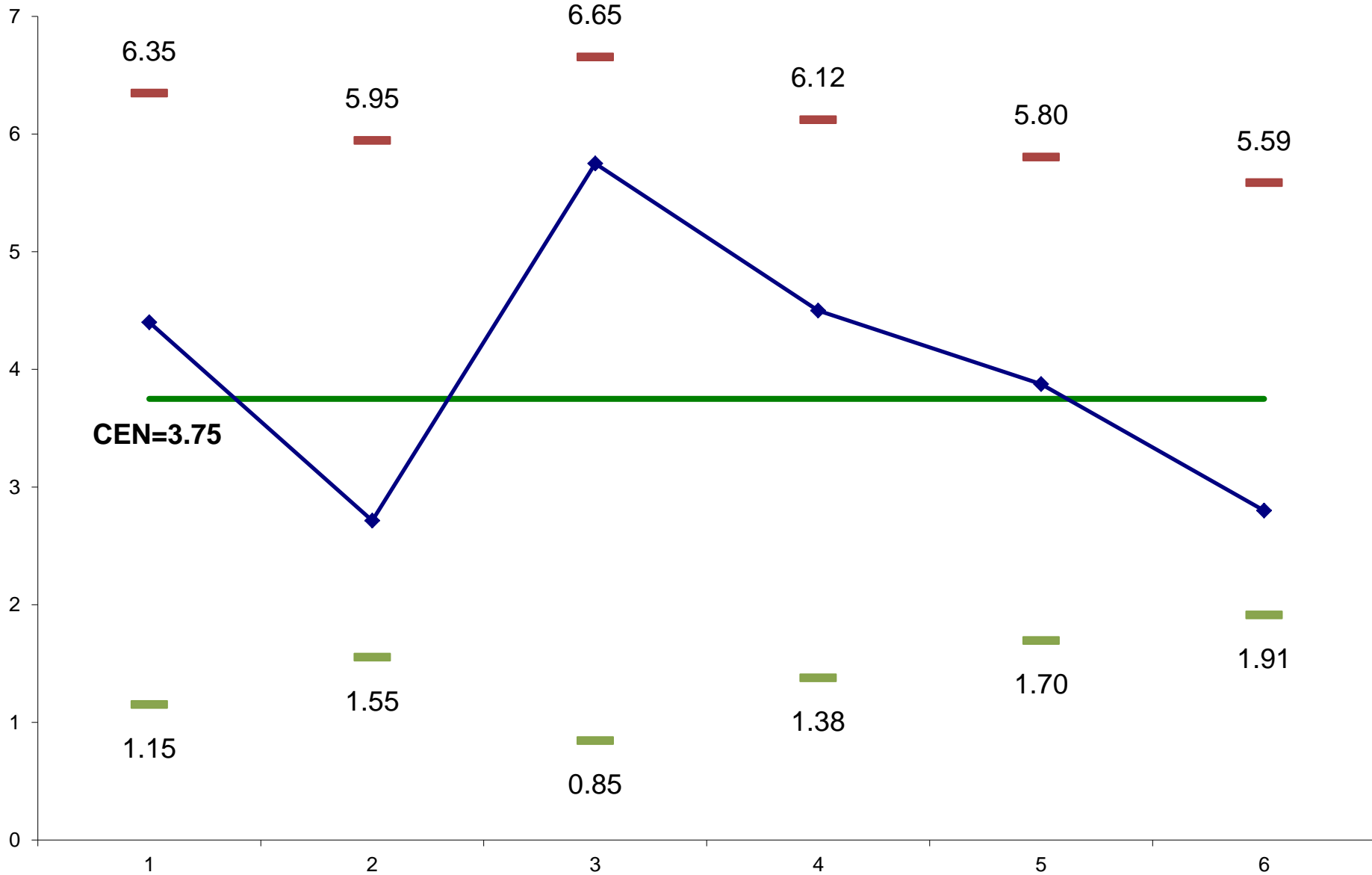
$$\bar{u} = \frac{\sum_{i=1}^k d_i}{N}$$

d_i – number of defects

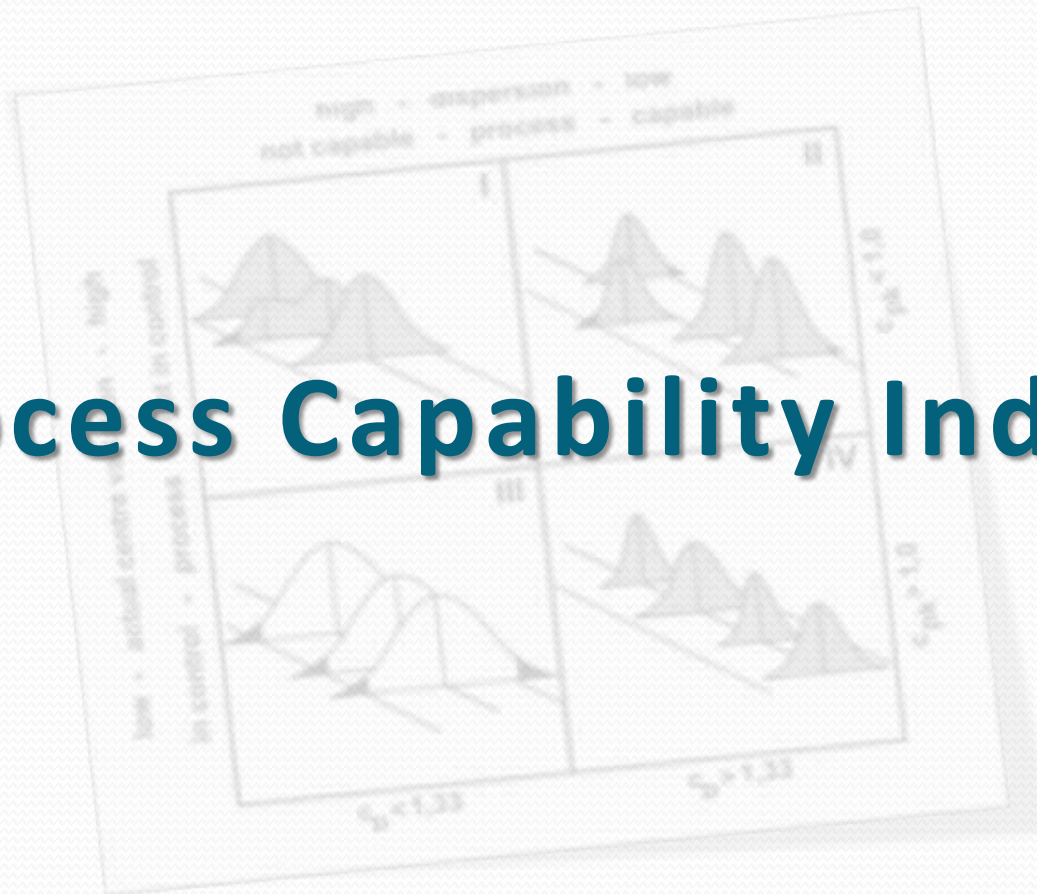
N – total number of sampled products in a series

$${}^U_L CL_{(\bar{u})} = \bar{u} \pm 3 \cdot \sqrt{\frac{\bar{u}}{n_i}}$$

u - Chart



Process Capability Indices



Process Capability Ratio C_p

- C_p - way of expressing process capability for a quality characteristic with both USL and LSL

$$C_p = \frac{USL - LSL}{6\sigma}$$

- If $C_p \geq 1.0$, the outputs of the process will be in accordance with the specification
- $C_p \leq 1.0$ the process yields products or services that are outside their allowable tolerance
- In general, the larger C_p is, the better

Example

- Calculate the process capability ratio for the production of additives used for the preparation of special polymers, which are used to protect rubber surface under extreme weather conditions.
- Specification: 9.0 ml \pm 0.5 ml
- Process mean $\bar{\bar{X}} = 8.80$ ml
- Standard deviation of the process $\sigma = 0.12$ ml

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{9.5 - 8.5}{6 \cdot 0.12} = 1.39$$

Process capability index - C_{pk}

- C_{pk} takes process centering into account unlike C_p

$$C_{pk} = \text{Min} \left[\frac{USL - \bar{X}}{3\sigma_o}, \frac{\bar{X} - LSL}{3\sigma_o} \right]$$

- Generally accepted minimum value for C_p is 1.33
- A C_{pk} of 2.0 means the process is capable of producing fewer than 3.4 defects per million (6 sigma process)
- Generally, if $C_p = C_{pk}$, the process is centered at the midpoint of the specification, and when $C_{pk} < C_p$ the process is off center

Example

- For the previous example, calculate the process capability index C_{pk}
- Specification: 9.0 ml \pm 0.5 ml
- Process mean $\bar{\bar{X}} = 8.80$ ml
- Standard deviation of the process $\sigma = 0.12$ ml

$$C_{pk} = \text{Min} \left[\frac{USL - \bar{\bar{X}}}{3\sigma_o}, \frac{\bar{\bar{X}} - LSL}{3\sigma_o} \right] = \text{Min} \left[\frac{9.50 - 8.80}{3 \cdot 0.12}, \frac{8.80 - 8.50}{3 \cdot 0.12} \right] =$$
$$= \text{Min} [1.94, 0.83] = \underline{0.83}$$

TAGUCHI'S QUALITY LOSS FUNCTION

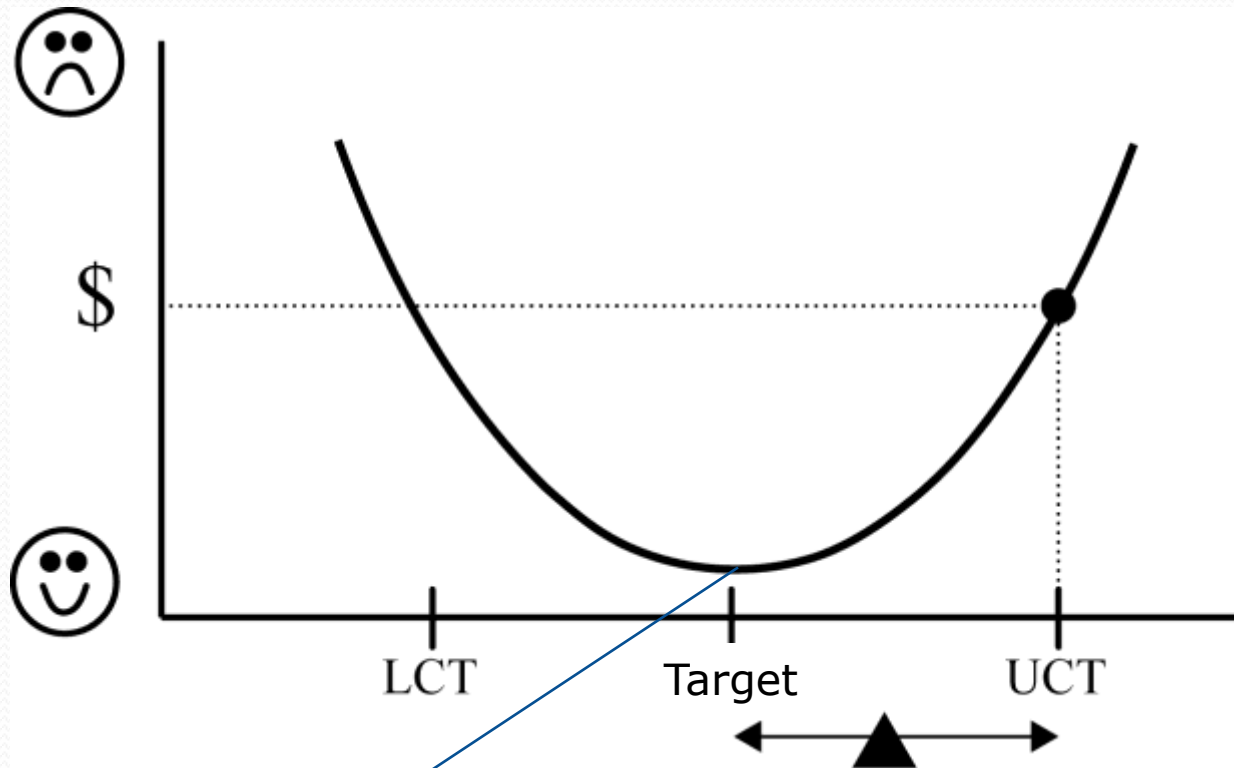


Taguchi principle



- In the traditional approach to the actual quality of the product was evaluated only in terms of respecting specifications
- It was believed that after sale of the product, the consumer was the one to bear costs due to quality loss
- Taguchi's associated customer satisfaction with the potential for "loss of reputation" associated with failure to meet customer expectations
- He changed the perspective of quality by correlating quality with direct and indirect costs not only at the manufacturing level, but also to the customer and society in general

Taguchi's Loss Function

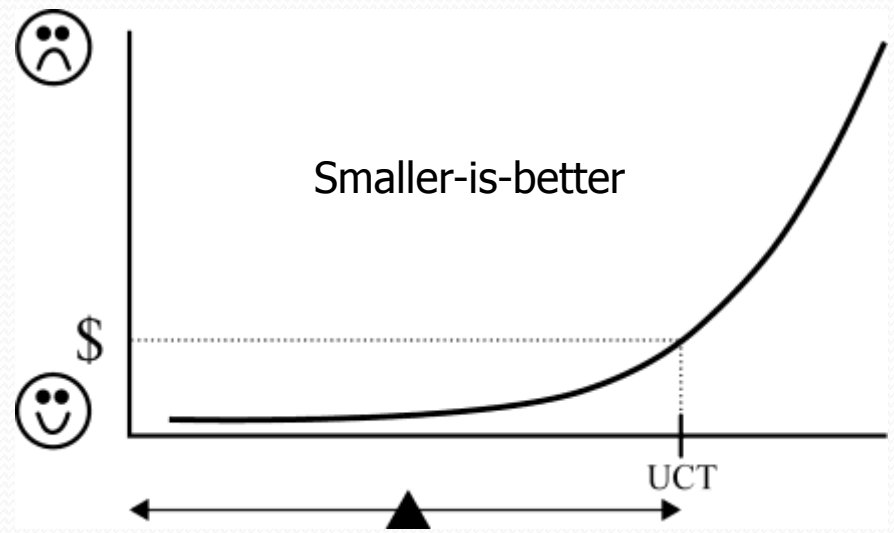
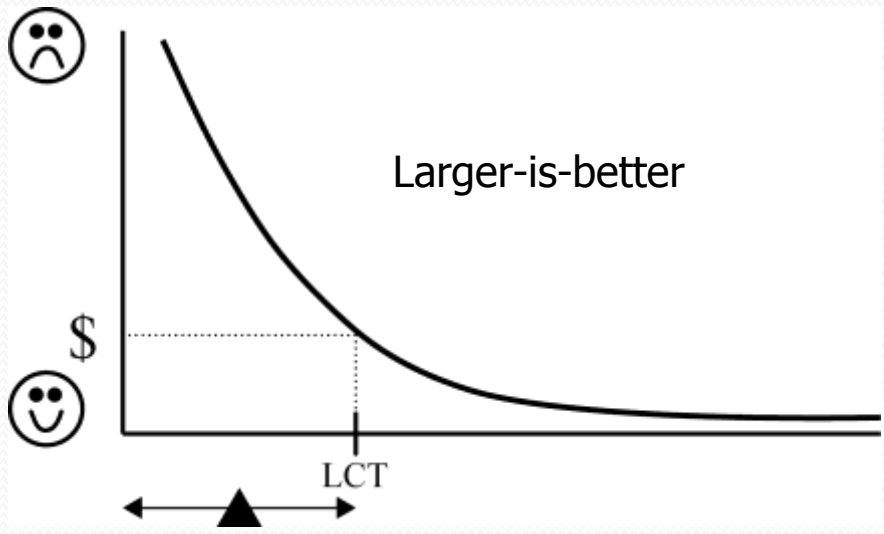


$$L(m) = 0$$

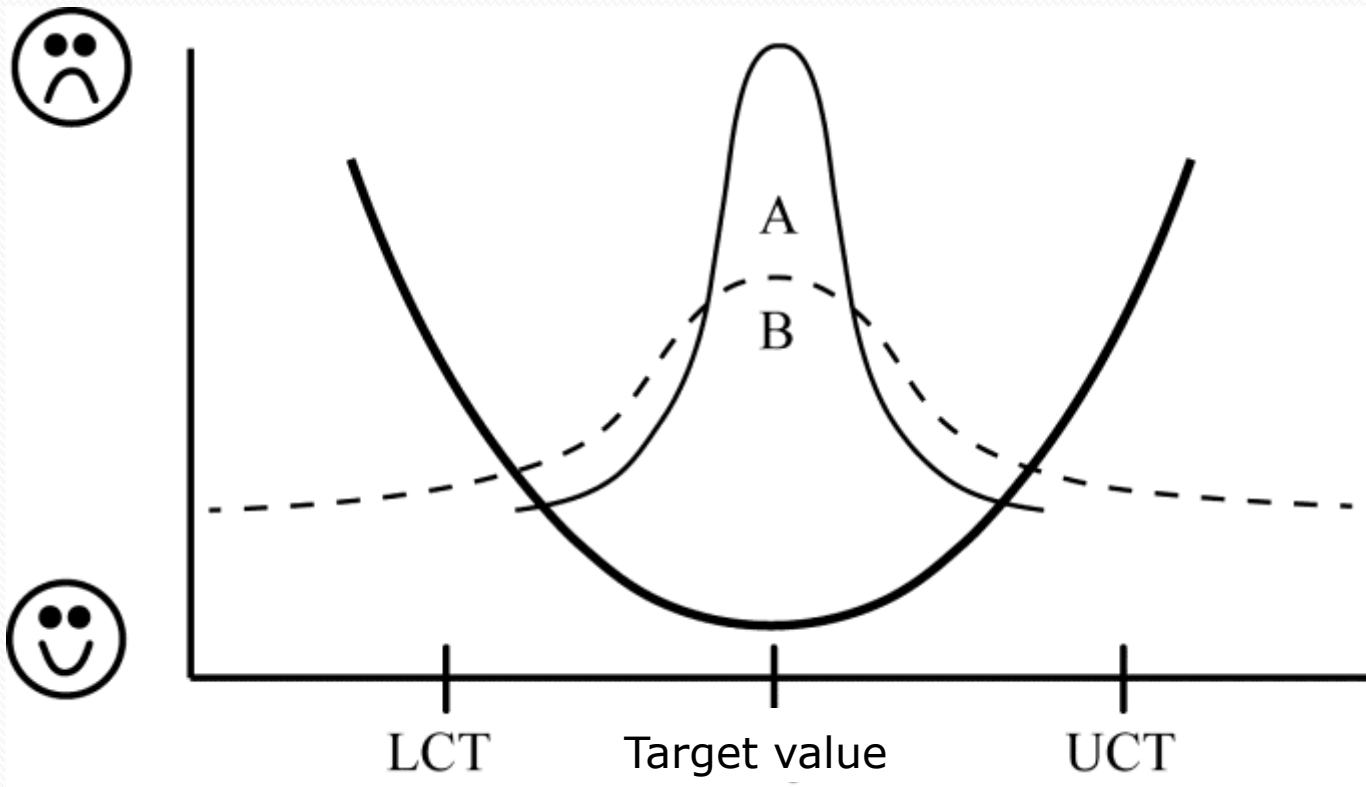


$$L(y) = k \cdot (y_i - m)^2$$

Characteristic curve shapes of loss function

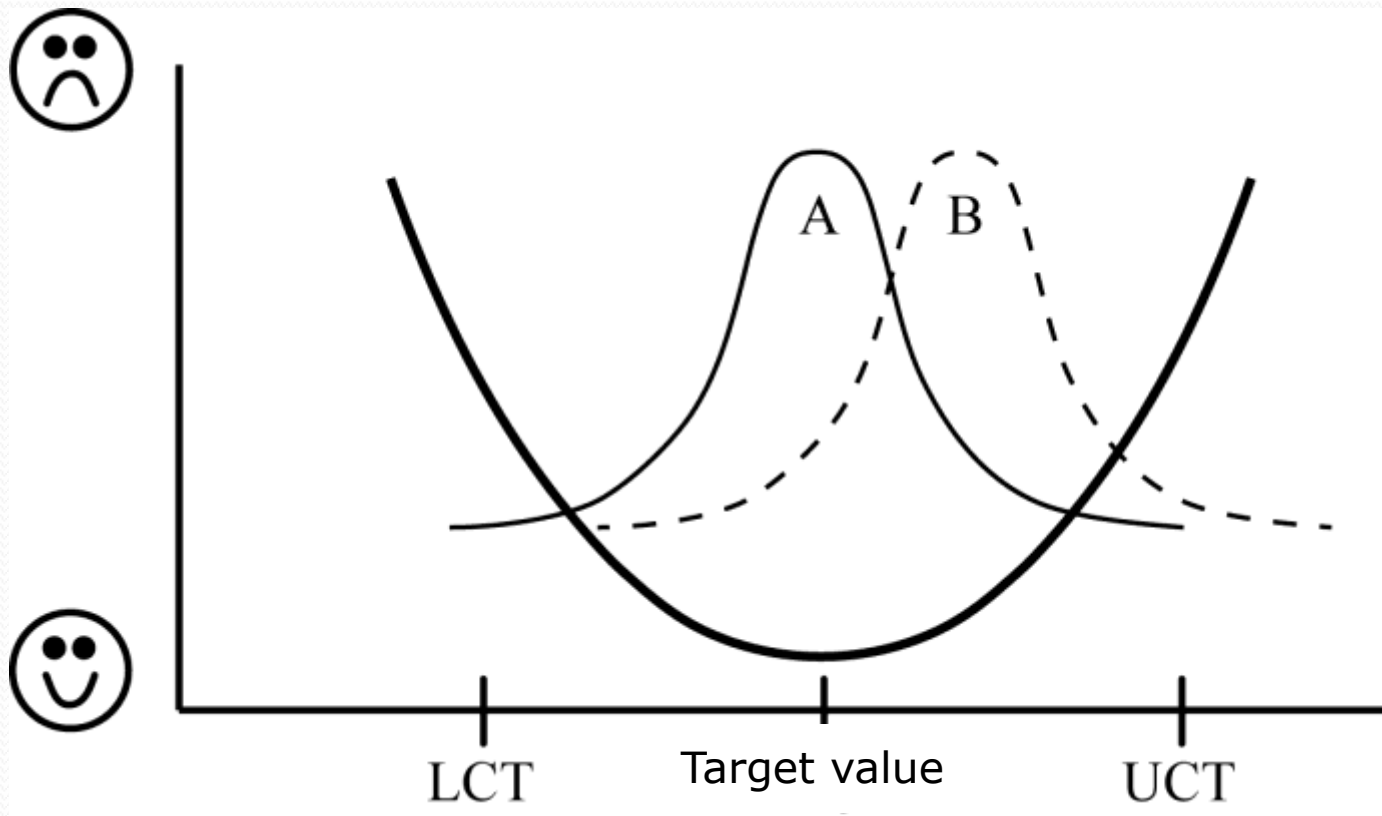


Dependence of the loss on the relationship between the average and variance



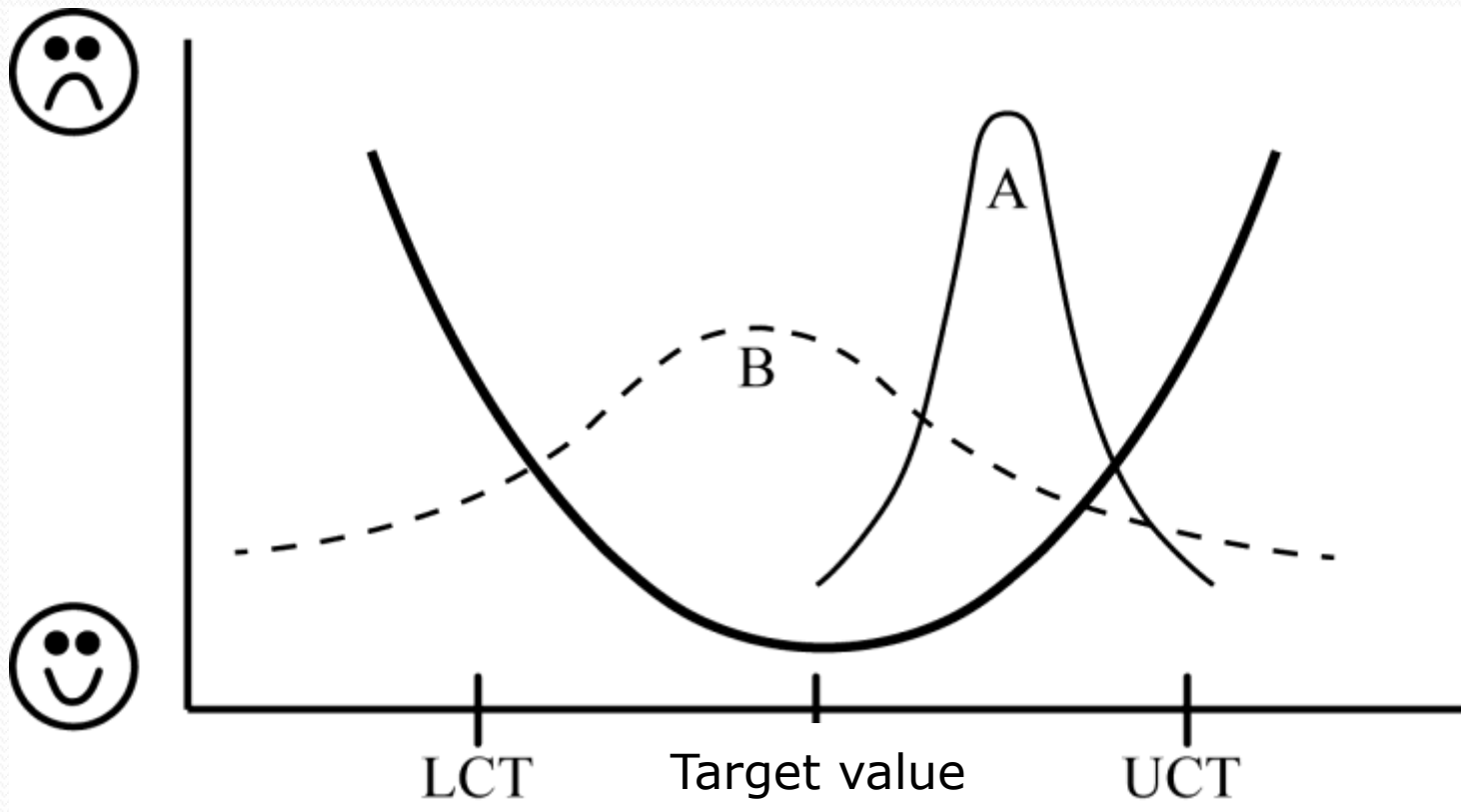
Products A and B have the same mean and different variances

Dependence of the loss on the relationship between the average and variance



Products A and B have the same variances and different means

Dependence of the loss on the relationship between the average and variance



Products A and B have different variances and means

Example

- Manufacturer of paints and varnishes wants to calculate losses related to the delivery of red paint to a car manufacturer. The target value is 100 g of pigment per 1 liter of the color. The average loss per customer which occurs due to the return of order is 600 euros. Quality of paint is considered unsatisfactory if the amount of the pigment exceeds the specification of $100\text{g} \pm 10\text{g}$. Calculate the loss that occurs due to poor quality, if a liter of paint contains only 85g of pigment.

$$L = k(y - m)^2$$

$$k = \frac{A_0}{\Delta_0^2} = \frac{600\text{€}}{10\text{g}^2} = 6 \text{ €/g}^2$$

$$A_0 = 600\text{€}; \Delta_0 = 10\text{g}$$

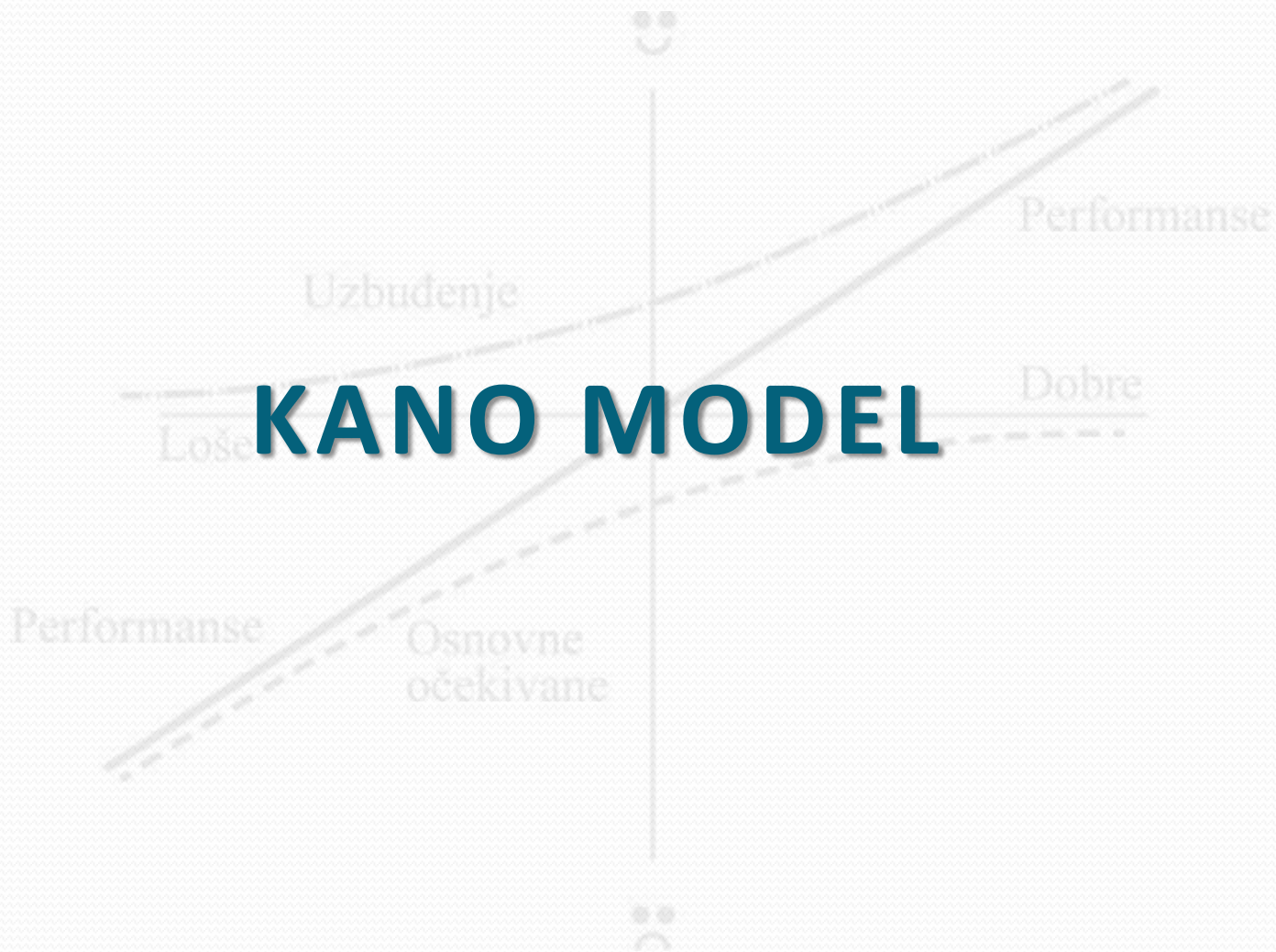
$$L = 6\text{€/g}^2 (85 - 100\text{g})^2$$

$$L = 1350\text{€}$$

A_0 – Average customer loss

Δ_0 – Functional tolerance

KANO MODEL



KANO MODEL

- Zero defect doesn't imply high customer satisfaction
- A company must identify and offer some unexpected features of its product or service which will positively surprise the customer
- Kano model is used to understand the importance of functions or features to a customer
- These functions or features of product or service are called needs

Customer needs

- Kano model sorts customer needs into one of three categories:

- **Basic needs**

The needs that are so fundamental that they are not expressed by the customer, except when we fail to fulfill them

- **Performance needs**

Meeting (or not meeting) these needs increases (decreases) customer satisfaction in proportion to the degree in which they are met (or not met)

Customer needs

- Kano model sorts customer needs into one of three categories:

- **Basic needs**

The needs that are so fundamental that they are not expressed by the customer, except when we fail to fulfill them

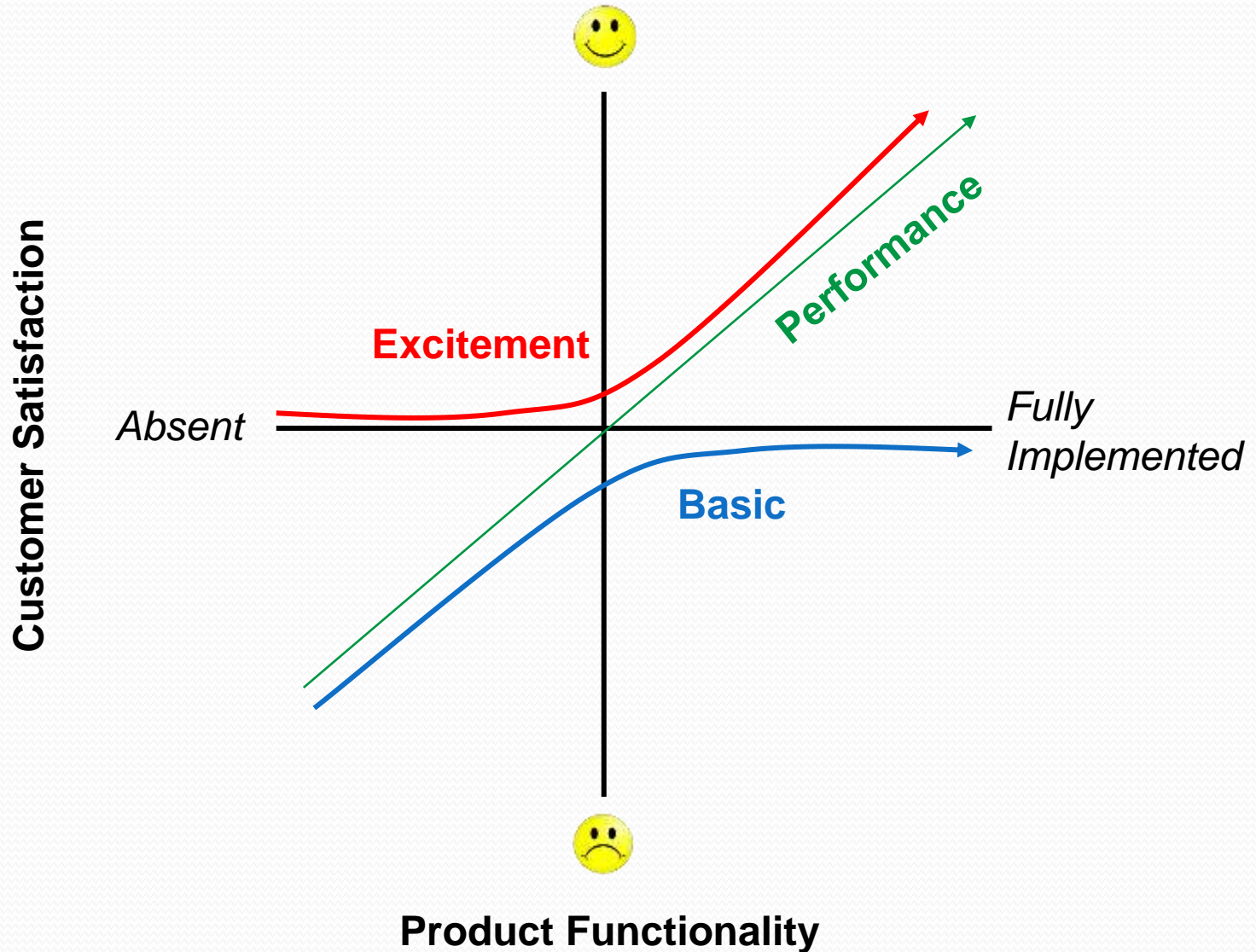
- **Performance needs**

Meeting (or not meeting) these needs increases (decreases) customer satisfaction in proportion to the degree in which they are met (or not met)

- **Excitement needs**

Difficult to identify because they are beyond the expectations of customers, their absence does not bring dissatisfaction among customers, but their presence immediately excites

Kano model



Kano's Paired Questions

- To determine the category of needs, Kano uses a set of paired questions
 1. question asks how you feel if something **exists**
 2. question asks how you feel if something **does not exist**

1. How do you feel if the instructor has a good sense of humor?

2. How do you feel if the instructor presents much useful information?

3. How do you feel if the instructor doesn't have a good sense of humor?

4. How do you feel if the instructor doesn't present much useful information?

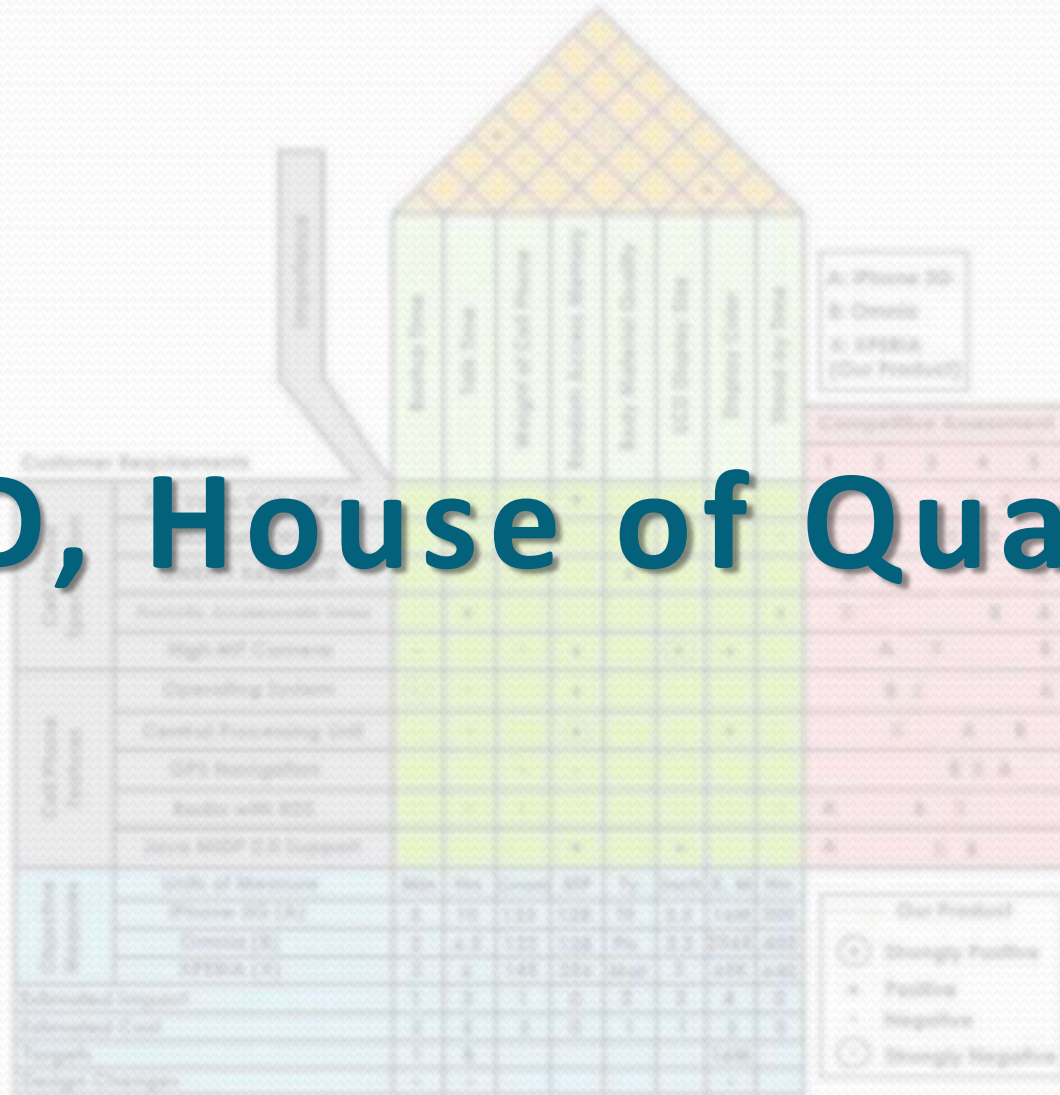
Use 1-5 to answer: 1. I really like it; 2. I like it; 3. I feel neutral; 4. I do not like it; 5. I really do not like it.

Use A-B to answer: A - I really like it; B - I like it; C - I feel neutral; D - I do not like it; E - I really do not like it.

The matrix for the selection of the category for needs

		POSITIVE		NEGATIVE		
		A	B	C	D	E
1	I really like it			<i>H</i>		
2	I like it					
3	I feel neutral	R				<i>e</i>
4	I do not like it	R				
5	I really do not like it	R	R	R		

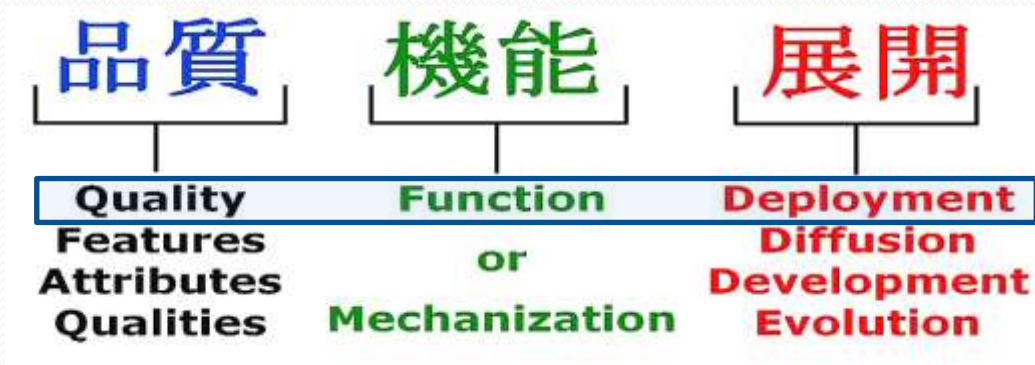
QFD, House of Quality



Our Product

- ⊕ Strongly Positive
- + Positive
- Negative
- ⊖ Strongly Negative

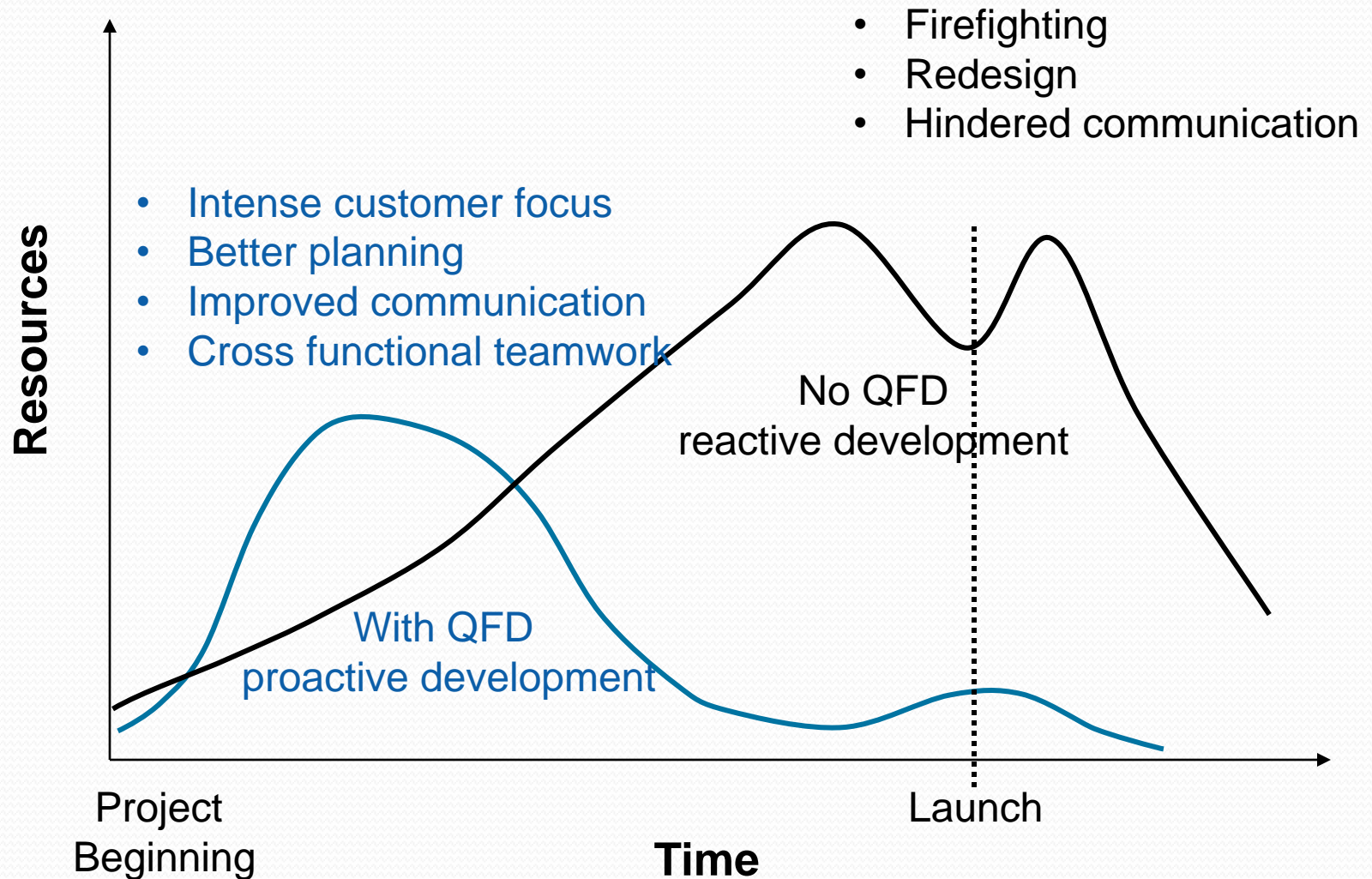
QFD



- Technique which allows the whole of the NPD process to be driven by the customer requirements
- “QFD is a way to assure the design quality while the product is still in the design stage” - Yoji Akao
- History QFD application:



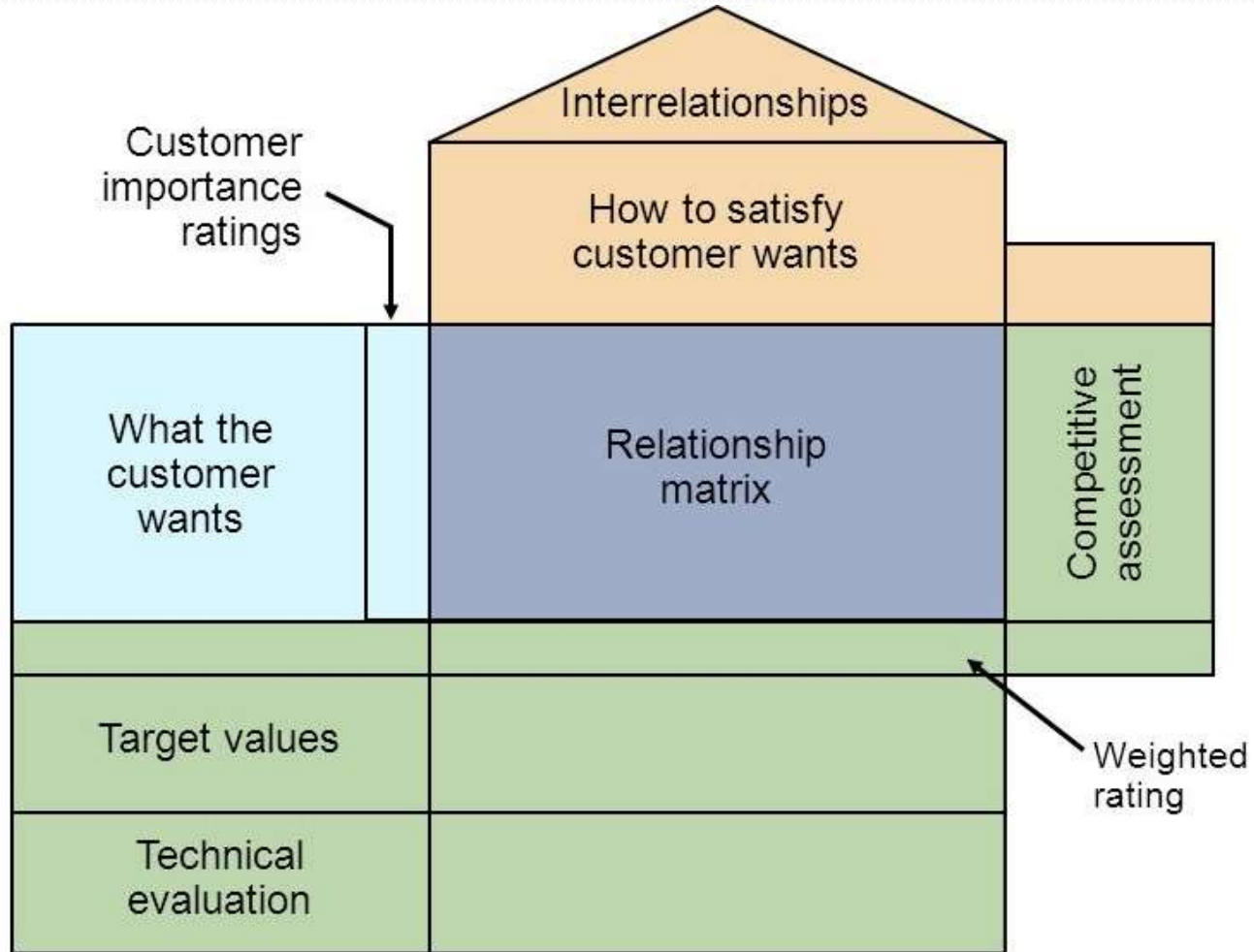
QFD vs. traditional development process



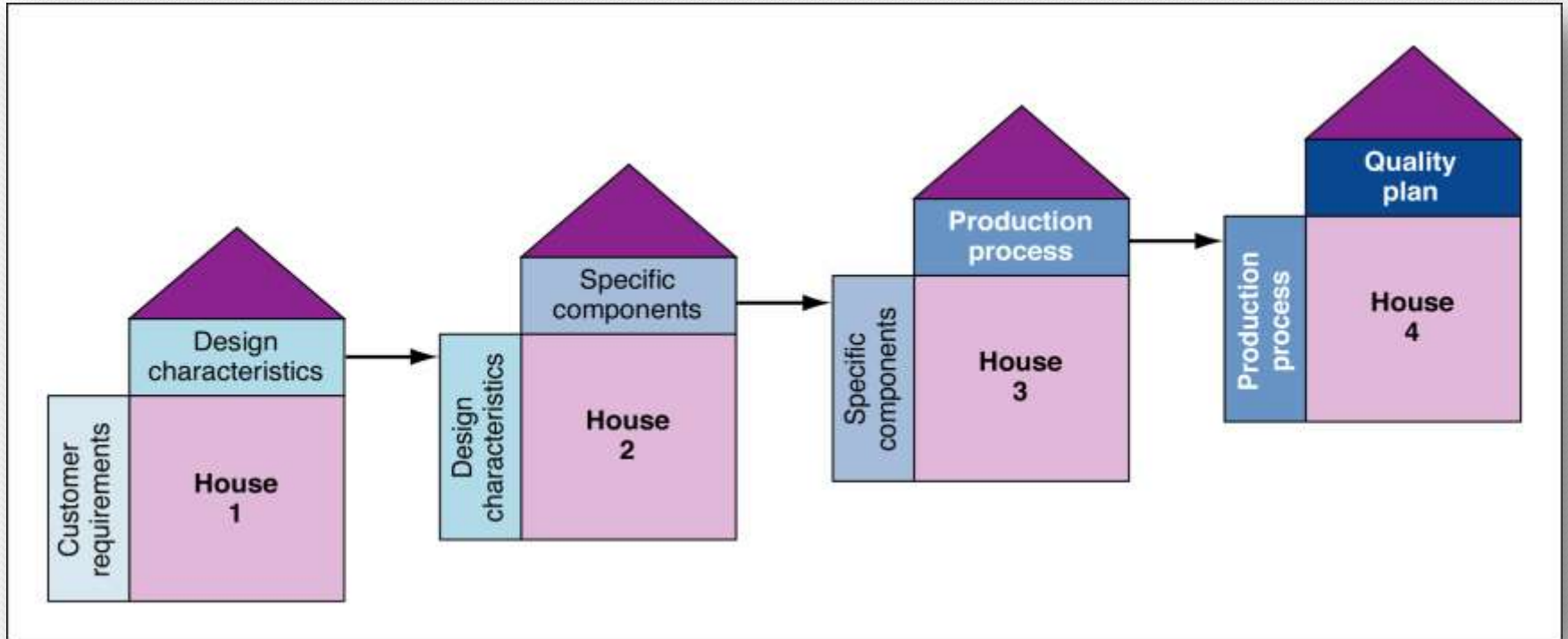
House of Quality

- Identify the customer
- Determine customer requirements (Kano model)
- Rank the requirements and determine their relative importance
- Translate customer requirements into measurable engineering requirements
- Competition benchmarking
- Develop importance ratings (Pareto analysis)

House of Quality



Houses of Quality sequence



House of Quality Example

